Improving Student Outcomes in Higher Education Mathematics

Joint statement from the Mathematical Association of America and the Conference Board of the Mathematical Sciences

The mathematical sciences community has implemented and studied new placement schemes and introductory curricula, together with student support systems, that are showing success at improving student success in postsecondary mathematics. Fully scaling such efforts in ways that expand this success will require further study to understand their critical components, as well as investments in support structures and faculty professional development. We believe that investments in such efforts, through the re-authorization of the HEA, will further the shared goal of improving student outcomes for our diverse student population.

Mathematics plays a central role in shaping student outcomes in higher education. Until the 1970s, most students did not take a math course unless their major required it. In the 1960s, only a few math department chairs at major public universities believed that mathematics should be part of a graduation requirement.[1] This changed over the ensuing decades. By 2010, most U.S. universities had a mathematics or quantitative reasoning requirement for graduation.[2]

As explained in the remainder of this document, the changing needs of our society with respect to mathematics education have created significant problems for higher education. At the same time, we know what needs to be done in terms of improved curricular options and better pedagogies to meet these challenges. Their uptake is accelerating, but is still slow. Faculty are reluctant to change the practices that worked for them even when these traditional approaches no longer meet the needs of the students they are supposed to serve. Math departments are often inadequately prepared and sustained for successful implementation of new curricula, pedagogies, and support structures.

The colleges and universities that have been at the forefront of these improvements have usually had discipline-based education specialists within the departments who led, studied, and adapted local interventions. Training and embedding specialists responsible for: i) implementing, studying, and adapting local innovations; ii) adapting assessments and curriculum to be more conceptually focused, and iii) supporting faculty to implement more engaging instruction constitute one of the most significant steps that could be undertaken with additional resources.[3]

We have a very small number of education specialists who are trained both in the mathematical sciences and in the research in undergraduate mathematics education. Expanding this corps and encouraging all departments to embrace their expertise would greatly facilitate the improvement of the mathematical experience for all students.

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[1] 15% according to Burdman et al (2018)
[2] 87% according to Schield (2010)
[3] See Chasteen and Code (2018) for details on the preparation and use of discipline-based education specialists.

Statement of the challenge

The National Assessment of Educational Progress (NAEP) reports that only about one-quarter of high school seniors are proficient in mathematics,[4] in line with ACT's assessment that only 40% of students who take the ACT demonstrate readiness for college mathematics.[5] The need for remediation in mathematics before taking any college-credit-bearing math course is widespread. Of students at public 2-year colleges, over half, 59%, take a remedial course in mathematics. Of the students who enter a remedial course in mathematics, only 21% will earn a degree within six years. The situation is better for students who are accepted into a public 4-year college, but it is still a third who enroll in remedial mathematics. Of these students, less than a third go on to earn a Bachelor's degree.[6]

Combined with other attrition, the six-year graduation rate of those who matriculate at a 2-year college (Associate or Bachelor's degree) is 48%, at 4-year colleges (Bachelor's degree) it is 59%. And the situation is worse for Black and Hispanic students where the six-year graduation rates are, respectively, 20% and 26% at 2-year colleges and 39% and 51% at 4-year colleges.[7] Our country cannot afford to lose all of this talent.

College completion is not the whole story. Mathematics is also the gateway to engineering and most scientific and technical careers. Even for those who arrive in college with mathematical proficiency, calculus is often a stumbling block. Nationally, over a quarter of those who enroll in the first mainstream calculus course will fail to earn a satisfactory grade. Another quarter will earn a C, a grade adequate for credit but a strong signal that further progress will be difficult. This is despite the fact that roughly 70% of those who enroll in calculus in college have already taken and passed it in high school.[8] It has been broadly reported[9] that even high performing calculus students complete calculus with weak understandings of fundamental ideas. Addressing this problem would enable more students to complete calculus with an A or B; concurrently enabling even the highest performing calculus students to acquire stronger mathematical conceptions and connections.

Given the dismal record of student performance in postsecondary mathematics, it is no wonder that in 2012 the President's Council of Advisors on Science and Technology castigated the mathematics community for the apparent lack of attention to this problem.[10] In fact, there had been and continues to be a great deal of attention paid to addressing these problems, but progress is slow and difficult when addressing the flaws in a system that was created over fifty years ago, has had its own noticeable successes, and has grown generations of adherents.

Beginning in the 1960s and accelerating through succeeding decades, the primary concern for departments of mathematics was to identify those students who were best prepared for careers in

[4] NCES (2015)
[5] ACT (2018)
[6] Chen (2016)
[7] Chen et al (2019)
[8] Bressoud et al (2015)
[9] Carlson and Rasmussen (2008)
[10] PCAST (2012)

engineering, science, or the mathematical sciences and to focus their attention on this select group. This is no longer sufficient. The demands of the 21st century have changed the requirements of our society. All Americans now need basic quantitative literacy to understand the issues facing us and to fully participate in civic society.

Alternate pathways and co-requisite models

Most students who require remediation are better served by taking a college class in statistics or quantitative reasoning. For over two decades, the Mathematical Association of America (MAA) has promoted quantitative reasoning as an alternative to college algebra.[11] The American Statistical Association has developed guidelines for statistical literacy for all students.[12]

Over the past decade, there has been a great deal of work on alternate pathways for students who enter college requiring remedial support. This was the subject of the recent National Academies workshop on *Increasing Student Success in Developmental Mathematics.*[13] Both the Carnegie Foundation for the Advancement of Teaching and the Charles A. Dana Center at the University of Texas, Austin have built pathways programs that focus on actual student needs. They have moved rates for successful completion of math requirements from less than one in five to over half.[14]

Another successful strategy is the use of co-requisites. This is just-in-time instruction and support for students who previously would have been placed into remediation. Co-requisite programs have been shown to equalize or even increase the odds of succeeding compared to the students who are normally admitted to the course. It has the added benefit that students do not increase the time to graduation. One example of the successful implementation of the co-requisite model is in the University System of Georgia where passing rates of 10–15% were increased to 60–70%.[15]

The importance of active-learning pedagogies

Those who think seriously about the needs of mathematics education for all students recognize that this requires embracing and supporting multiple pathways through mathematics, both diverging and frequently reconnecting, opening routes into scientific or technical careers for non-traditional students. More careers than ever before presuppose a degree of mathematical competence, and this competence is now recognized as encompassing a far broader landscape. Statistics, data science, operations research, and computational science multiply the directions in which students might pursue mathematics. In addition, we can no longer afford to focus exclusively on conventional sources of engineers and scientists. To maintain the needed numbers, we must pay attention to how we support and encourage all students, and we must devise new interventions for attracting more women and students from traditionally underrepresented groups.

[11] Steen (2001)
[12] Franklin et al (2007)
[13] See https://sites.nationalacademies.org/DBASSE/BOSE/Developmental_Math/index.htm
[14] Huang (2018)
[15] Denley (2019)

Since the 1980s, the mathematical community has recognized that needed changes require more than new curricular paths. They require a new mindset for how we teach. Traditional lecture methods have been adequate for an elite group of students who arrive in college equipped to build a personal understanding of the mathematics they study. These are mostly the children of college-educated parents and the students who have attended our best high schools. Traditional methods of instruction have further separated the haves from the have-nots. A student who enrolls in a public 4-year undergraduate program and whose parents have not gone beyond high school has only a 38% chance of earning a Bachelor's degree. If either parent has completed a Bachelor's degree, the likelihood of graduation almost doubles to 74%.[16] We have become very good at giving an excellent education to our most privileged students.

What we have long suspected and have verified in recent years is that moving to a student-centered, active-learning pedagogy can greatly improve the success of our weakest students while enhancing the learning and mathematical experiences of the strongest.[17] The evidence is now so clear that the presidents of the professional societies in the mathematical sciences have all signed onto a joint statement requesting the investment of "time and resources to ensure that effective active learning is incorporated into postsecondary mathematics classrooms."[18]

This approach helps all students. Active-learning pedagogies are especially important in the preparation of preservice teachers. They help to ensure that these prospective teachers have strong understandings of foundational ideas of the mathematics they will need to draw upon in their teaching. They also equip these teachers to engage their own students in the development of mathematical ways of thinking, thus dramatically improving student preparation for postsecondary mathematics.

The leadership role of the professional societies in the mathematical sciences

Over the past decade MAA has continued its leadership role in identifying and supporting the implementation of best practices in undergraduate mathematics education. It led the creation of the joint report, *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025*,[19] describing the need for updated curricula, well-articulated pathways, evidence-based pedagogical methods, the removal of barriers at critical transition points, and strong connections with other disciplines. The MAA's national studies of calculus instruction[20] have provided insight into best practices with respect to placement, student support services, construction of engaging curricula, coordination of instruction, training of graduate teaching assistants, use of data to guide curricular and structural modifications, as well as the use of active-learning strategies. Through its *Instructional Practices Guide*,[21] MAA has publicized practical steps to

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[16] Chen et al (2019)
[17] Laursen et al, (2014) and (2016). Freeman et al (2014)
[18] CBMS (2016)
[19] Saxe and Braddy (2015)
[20] Characteristics of Successful Programs in College Calculus, NSF #0910240, and Progress through Calculus, NSF #1430540
[21] MAA (2017)

improving classroom practices, assessment, and course design. Its *Curriculum Guide*[22] provides recommendations on what is taught as well as dealing with recruitment, retention, articulation, placement, and preparation for graduate study.

Many of the professional societies in the mathematical sciences have recently released reports that describe best practices. The American Statistical Association's *Guidelines for Assessment and Instruction in Statistics Education College Report*[23] provides guidance into what should be taught in college-level statistics and how it should be taught. *Guidelines for Assessment and Instruction in Mathematical Modeling Education*[24] explains how to educate students at both K-12 and postsecondary levels in the aspect of mathematics that is most frequently needed both in the workplace and in society in general, the ability to understand, create, and analyze mathematical models. The American Mathematical Association of Two-Year Colleges has published its recommendations for effective instruction in two-year colleges, *Improving Mathematical Prowess and College Teaching*.[25] *Standards for Preparing Teachers of Mathematics*[26] is a product of the Association of Mathematics Teacher Educators, laying out expectations for high-quality teacher preparation.

Research and the experiences of implementation have demonstrated what works to strengthen the preparation of our scientific and technical workforce while equalizing opportunities for all of our students. There is now an abundance of evidence that all students would benefit from experiencing more meaningful and coherent instruction from the students' perspective—that is, mathematics curriculum and instruction that is more engaging and focused on developing students' mathematical understandings and practices. The professional societies in the mathematical sciences, supported by college and university consortia, are working toward the wide adoption of these best practices.

About the Mathematical Association of America: The mission of the MAA, founded in 1915, is to advance the understanding of mathematics and its impact on our world. The MAA has long been a leader in providing guidelines for the undergraduate program in mathematics, as well as professional development for improvement of all aspects of postsecondary mathematics.

About the Conference Board of the Mathematical Sciences: CBMS is an umbrella organization consisting of eighteen professional societies all of which have as one of their primary objectives the increase or diffusion of knowledge in one or more of the mathematical sciences. Its purpose is to promote understanding and cooperation among these national organizations so that they work together and support each other in their efforts.

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[22] Zorn (2015)
[23] GAISE College Report ASA Revision Committee (2019)
[24] Garfunkel and Montgomery (2019)
[25] AMATYC (2018)
[26] AMTE (2017)

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