

CBMS Statement on the Need for a Grades 9-14 Mathematical and Statistical Sciences Framework

Overview

The Conference Board of the Mathematical Sciences (CBMS) exists to promote understanding and cooperation among professional membership organizations whose primary objectives are focused in one or more of the mathematical sciences. CBMS, as an umbrella organization, occupies a unique position to: bring together diverse perspectives; find common ground on issues around research, education, and the uses of the mathematical sciences; and convene groups to move mathematics education forward.

It has become clear to the CBMS member organizations that there is an urgency to address evolving needs in Grades 9-14 mathematics education. This work will require the cooperation of wide-ranging stakeholder groups, including many outside CBMS. This statement calls for the new Mathematical Sciences Education Board (MSEB) at the National Academies to prioritize a consensus study to create a Grades 9-14 mathematical and statistical sciences framework. Such a document could serve as a guide for the development of programs around the country and provide a baseline for consistency. To that end, this CBMS statement: outlines the current situation; highlights existing agreements and needs in content, pedagogy, and technology; and provides a sample charge to the MSEB.

Introduction

The growth of the mathematical and statistical sciences and the increasing scope of their applications in the first decades of the 21st century have been breathtaking. The National Academies volume, *The Mathematical Sciences in 2025*, published a decade ago, anticipated that the influence of big data, artificial intelligence, and other new technologies would expand the demands and opportunities in the mathematical and statistical sciences (National Research Council, 2013). This has proved prescient as their impact already has surpassed what many could have imagined even just a few years ago. The nation's future demands that the learning experiences of all students should afford opportunities for them to flourish mathematically (White House Office of Science and Technology Policy, 2024). Mathematical experiences should support all students in reaching their academic goals and personal aspirations, support economic prosperity, and sustain an informed society. The risk that many students will miss these opportunities is simply too great to ignore.

In response to the exciting developments in the mathematical and statistical sciences, institutions of higher education have created or modified mathematics and statistics courses to better meet the needs of students across different majors. Students in many institutions now experience mathematics and statistics that is more closely aligned to their needs and interests.

There is, however, a significant lack of consistency in how these changes have been implemented around the country. This in turn sends mixed signals to secondary institutions about how best to prepare students in the mathematical and statistical sciences. The mathematical prerequisites for new courses vary and may not match longstanding curricula. High schools, which must simultaneously prepare their students for multiple post-secondary institutions and other career options, must now navigate a world in which policies and curricular options vary by state, and sometimes even by district. Secondary institutions, in revising their curriculum to better meet student needs, must also be mindful to keep doors open to relevant post-secondary options. This potentially poses a difficult conundrum: keep the status quo as the “safe” route, blind to the dramatic growth in the mathematical sciences and discourage many students from future study, or update mathematics offerings while potentially limiting future opportunities due to differences in existing structures.

Around the country, states are working to tackle these issues. The approaches in different states vary widely: each state brings its own sets of policies, landscapes, and issues that require careful planning. The individual state curricula and policies make it challenging for states to work together to meet these challenges. While some of the early solutions are individually excellent, having each state go its own way is not an ideal approach and may not keep paths open for all students, especially for those who may move from one state to another.

In view of these concerns, the Council of the Conference Board of the Mathematical Sciences (CBMS) recognizes that now is the time to empower a healthy, nationwide conversation around updating mathematical and statistical sciences in secondary and early post-secondary programs to best meet these opportunities and challenges.

We believe that a newly constituted Mathematical Sciences Education Board (MSEB) at the National Academies is an ideal entity to commission a consensus study to develop a framework that provides guidance for Grades 9-14 mathematics and statistics. Such a report would have far-reaching consequences and serve as a reference for many years. A new framework could address issues where unhealthy debate too often occurs. These issues include content, pedagogy, technology, and how they interact. In the following sections, we clarify some of the key points around these issues while also clarifying the need for a field-level framework to guide leaders as they engage in the work of improving mathematics and statistics education in Grades 9-14.

Content

A framework for Grades 9-14 content in the mathematical and statistical sciences should focus on the power of mathematical and statistical thinking and promote the equal standing and integration of: reasoning with quantities and their relationships; reasoning with uncertainty and data; computational reasoning; abstract reasoning through generalizing, specifying, and inference; and model-based reasoning (cf. OECD, 2022). These ways of reasoning are each distinct, can complement each other, and are indispensable in modern problem solving (e.g., Pollak, 2015; Son & Stigler, January 25, 2023) at the secondary level and beyond, supporting understanding and use of calculus, linear algebra, statistics, geometry, and more. Across

mathematics and statistics and beyond, an “ability to reason logically and to present arguments in honest and convincing ways is a skill that is becoming increasingly important in today’s world” (OECD, 2022, p. 15). Expertise in each type of reasoning includes constructing, interpreting, evaluating, and communicating ideas.

- **Reasoning with quantities and their relationships** refers to representing quantitative information and then using these representations to analyze properties of quantities and their relationships. Reasoning with quantities involves understanding scale, unit, and measurement in context. Solving problems with quantities may involve comparing, estimating, and checking estimates for plausibility; using proportional, additive, multiplicative, or exponential reasoning; or recognizing the limitations of any particular method for computation. “To use quantification efficiently, one has to be able to apply not just numbers, but the number systems ... what makes them ... a powerful tool are the operations” (OECD, 2022, p. 16). Students must also learn to attend to two or more simultaneously varying quantities, coordinate the ways they change with each other, and interpret these relationships in context (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). Quantities and their rates of change may be represented in a variety of ways such as graphs, tables, figures, relations among variables, and parametric equations. Reasoning with relationships among quantities supports understanding of rates of change and accumulations (Confrey & Smith, 1995) and it allows students to work with simultaneously varying quantities across continuous domains (Thompson & Carlson, 2017). Reasoning with relationships among quantities is needed to understand functions and relations from physical or social phenomena through mathematical and statistical perspectives.
- **Reasoning with uncertainty and data** relies on using and analyzing data to drive discovery and inform decisions. It involves the ability to identify raw data of all types, to ask how the data were produced, and to understand how variable transformations can produce different representations of the same data (American Statistical Association [ASA], 2020). All students should acquire data acumen, be able to carry out a problem-solving process that is investigative, and understand their responsibility to be ethical practitioners. They should be able to formulate questions which anticipate variability, collect or consider data within the given context, and offer analysis and interpretation that recognize and quantify the variability. Students should develop a sense of when they are working with randomness and uncertainty, that is, where a situation is probabilistic rather than deterministic. Sources for probabilities may be subjective (such as persons’ underlying beliefs) or objective (such as rolling dice). Students must be able to deal with the fast-changing nature of data and the tools required for analysis.
- **Computational reasoning** refers to the thought processes involved in expressing solutions or aspects of solutions as computational steps or algorithms that can be carried out by a machine, and to recognize modularity in potential solution approaches (Stephens & Kadijevich, 2020). Computational reasoning spans capabilities to create models and simulations to illustrate relationships, analyzing the results of existing computational models, and evaluating models (National Research Council, 2012). When students develop computational reasoning, they become more able to modularize how to work with complex tasks involving quantification, integrate various components to cohere across assumptions or constraints, reformulate design principles in the face of analysis,

and assess simulations for not just correctness or efficiency but also for simplicity and design (Wing, 2006).

- **Abstract reasoning through generalizing, specifying, and inference** refers to processes for inquiry about patterns and relationships; creating and understanding structures; inferring structural properties; and developing and communicating deductive reasoning. These processes support conceptual understanding by helping a community understand when a pattern or relationship holds, why and under what circumstances algorithms and procedures work, and why symbolic representation may be sensible. Inference involves developing argumentation through layers of deductive and inductive reasoning. Communicating argumentation can be done in a variety of ways, though in all cases, it should be a sequence of valid assertions that may be communicated to the relevant community (Ball & Bass, 2003; Stylianides, 2007). The skills and thinking involved in this kind of argumentation also can be later applied to building and explaining mathematical models, understanding comparisons of the power of different statistical methods, and analyzing the efficiency of different computational algorithms.
- **Model-based reasoning** involves constructing conceptual, mathematical, or statistical frameworks that capture the essential features of a system being studied (e.g., Consortium for Mathematics and Its Applications [COMAP] & Society for Industrial and Applied Mathematics [SIAM], 2019; Lesh & Zawojewski, 2007; Pollak, 1979). Examples of model-based reasoning include using trigonometry and exponential functions to approximate temperature data (e.g., Exploratorium, n.d.), estimating piano tuners in a particular city, using students' measurements and comparisons of quantities to develop variables, developing the number line and arithmetic properties from students' experiences of a subway ride (e.g., Moses & Cobb, 2001), and even information theory and Turing machines. Model-based reasoning depends on mathematizing, where students carefully articulate assumptions and definitions, so as to communicate features of systems with precision that ultimately support symbolic representation and reasoning (Moses, Kamii, Swap, & Howard, 1989). Model-based reasoning helps us understand phenomena, test theories, refine knowledge, predict future behavior, and optimize systems.

The CBMS Council members suggest that the framework supports these ways of reasoning across content and course offerings. Additionally, we suggest that the framework provide a resource for considering how content is vertically articulated and integrated across Grades 9-14. That is, ways of reasoning emphasized from the beginning to the end of high school should support and be intimately aligned with post-secondary education. Students should see connections across different disciplines of the mathematical and statistical sciences and connections to science, social science, and engineering throughout their education. A variety of modes of reasoning in the mathematical sciences are needed for equitable participation, no matter the ultimate path that students take.

In addition to a focus on the above modes of reasoning, there is a growing need for guidance on determining the mathematical content and experiences that all students in high school need, and where branching of content might occur in ways that build strength in the above ways of reasoning, in equitable ways, and for the opportunities available after when branching occurs.

Pedagogy

There is generally widespread agreement on what high quality mathematics teaching and learning look like. In fact, numerous mathematics and mathematics education organizations have advocated for mathematics instruction that includes engaging students in tasks that promote mathematical investigation, sense-making and problem solving; engaging in communication that builds individual and collective mathematical competencies; and building procedural fluency from conceptual understanding (AMATYC, 2018; CBMS, 2016; ASA, 2020, Mathematical Association of America [MAA], 2018a; NCTM, 2014). Moreover, equitable teaching practices, including focusing on mathematical thinking, capitalizing on students' cultural and linguistic funds of knowledge, and addressing issues of power and social justice, cultivate all students' participation and positive identity (e.g., Aguirre et al. 2024; Apkarian et al., 2024; Battey, 2013; Walshaw, 2010; Zavala & Aguirre, 2024). Equitable teaching practices depend on both effective teaching practices and an attention to who participates in the classroom; over time, equity in classrooms has the potential to foster equity in society (e.g., CBMS, 2021).

Yet there are still too many students, at all levels, who may not be given equitable opportunities to participate mathematically (Hanushek et al., 2022; Johnson et al., 2020; Melhuish et al., 2022; Reinholz et al., 2022). All students must have frequent opportunities to be actively engaged in reasoning and sense-making. Such opportunities support seeing math not as a set of rules to be memorized but as conceptually coherent, and as an authentic activity that offers the benefits of better understanding and improved navigation of their world. Engaging in reasoning and sense-making includes activities such as focusing on, looking for, and making sense of patterns, taking complex problems and decomposing them into ones easier to resolve, determining appropriate strategies to utilize, and addressing problems logically. As part of these activities, students must be supported to clearly and precisely communicate their thinking with appropriate mathematical language (Charles A. Dana Center, 2019).

Recent discussions about effective pedagogy have included several mischaracterizations and an either/or mentality that are not productive as we as a community work to improve mathematics teaching and learning. For example, active learning and direct instruction have been characterized as polar opposites and incompatible approaches in a classroom, and conceptual understanding and procedural fluency have been positioned as a choice—valuing one devalues the other. There are also competing understandings of what “rigor” means at the K-12 level (Charles A. Dana Center, 2019), with many adopting a narrow view of rigor as proving theorems or working on difficult problems. Rather than focusing on positioning these ideas against one another, as practices that must be chosen between, we must instead consider when and how a variety of instructional practices and foci are best used to support student learning.

Despite wide-spread agreement on the most effective mathematics teaching practices, implementation of these practices has proven to be challenging. In fact, many changes to teaching practice that support meaningful learning were proposed over 30 years ago (NCTM, 1991), but are still not occurring as widely or quickly as desired. Perhaps too much attention has

been given to covering the content and not enough to engaging students in authentic mathematical practices and more broadly developing the mathematical and statistical habits of mind of professionals who use and apply mathematics every day. Thus, additional attention should be paid to types of experiences students are having in mathematics classrooms. Focusing on these practices and the experiences they afford students in their mathematics learning, rather than solely on content, may help accelerate the pace of change in implementing effective mathematics teaching practices.

Additional guidance is needed to help educators make changes to their practice that will provide students with opportunities to engage in authentic mathematical work. For example, providing a framework for the sorts of experiences that all students should have, along with guidance for which mathematical and statistical practices and habits of mind best align with the learning of different mathematical or statistical content, would be a powerful tool for teachers. An alignment document and meaningful examples could serve as a consistent reminder for educators to focus on engaging students in those practices and experiences. A matrix that matches content with the mathematical and statistical practices, as well as the ways of reasoning described in the previous section, could also aid educators in determining how best to assess students on both their understanding of the concepts and their ability to engage in authentic mathematical practices. With such guidance, assessment might continue to shift from overly focusing on procedures to focusing on those mathematical and statistical practices and habits that students should be developing.

Technology

Technology used appropriately helps students make sense of ideas and can be used to solve challenging, sophisticated problems. It enables students to experience mathematics through experimentation, visualization, sense-making, reflection, and discussion. As such, the CBMS Council members believe that all Grades 9-14 students should experience and gain confidence with a variety of technological tools and be able to generate, validate, and verify solutions provided for a wide range of problems (AMATYC, 2018, p. 3; AMTE, 2022; NCTM, 2023; MAA, 2018).

Much of the current pedagogical delivery emphasizes traditional analytical solutions, limiting problems to be less sophisticated than what can be tackled with current digital tools. However, teachers who use technology strategically can provide all students greater access to mathematics (AMTE, 2022; NCTM, 2023). The incorporation of digital technology into the classroom environment can no longer be viewed as a discretionary pedagogical choice, as reluctance to use technology in the classroom limits the ability of students to conceptualize and advance their understanding of mathematics. It should enhance, not replace, human interaction and become as foundational as the use of textbooks, paper, and pencil in Grades 9-14 classrooms. Technology use should extend beyond calculators to include powerful software environments that are integrated into the curriculum in a deliberate and consistent manner.

Yet there is currently no widely agreed upon guidance for incorporating technology into the mathematics curriculum. The framework should provide a series of recommendations for

pathways to meaningfully integrate technology into the learning environment. It could do this by addressing appropriate technology integration, such as how to leverage technology to explore and validate solutions. It could also connect technology to particular content and ways of reasoning, provide criteria about when to use technology and when not to use technology, and highlight how not using technology puts students at a disadvantage. It would also ideally solicit input from a variety of disciplines and agencies, providing a holistic approach and highlighting opportunities to synthesize content areas to better prepare students to tackle interdisciplinary challenges.

Beyond the existing uneven implementation of technology tools in the classrooms, overall access to technology in schools varies widely. The Digital Access Divide (U.S. Department of Education, n.d.) is a barrier that stands between students and educators in terms of equitable access to high-speed internet, hardware, and digital resources. While the framework cannot solve this issue, it might seek to provide guidance on possible ways to diminish that divide.

Sample charge for a 9-14 Mathematical and Statistical Sciences framework

The National Academies of Science, Engineering, and Medicine will convene an *ad hoc* committee to develop and define a framework to guide the goals and design of mathematics education in Grades 9-14. In preparing its report, the committee will draw on current research and evaluation in mathematics teaching and learning from both qualitative and quantitative traditions; expertise from professionals in the mathematical sciences and partner disciplines, including mathematicians, statisticians, education researchers, and teachers; and career and technical educators and industry professionals; and with representation from those who work across Grades 9-14 in secondary education and post-secondary education including high schools, community colleges, 4-year colleges, and universities. The committee's report will articulate:

- Ways of reasoning that all students should have the opportunity to develop in Grades 9-14 mathematical and statistical sciences. These may include reasoning with quantities and their relationships; reasoning with uncertainty and data; computational reasoning; abstract reasoning through generalizing, specifying, and inference; and model-based reasoning. (*Guiding questions: What does it mean to learn mathematics? What should be the learning goals of Grades 9-14 mathematics?*)
- Experiences that all students should have with respect to content. (*Guiding question: What core content should have prominence?*)
- Potential pathways (branches) and the mathematics needed in each, including guardrails for state pathways initiatives. (*Guiding questions: What is the mathematics that all students need before branching into specialized content? How could that content be organized in potentially modularized ways?*)
- Pedagogies applicable to Grades 9-14 mathematics. (*Guiding questions: What debates can we put to rest? What does balance look like? Which pedagogies work effectively in which cases?*)
- Technologies that students should have access to and when. (*Guiding questions: What technologies should students be expected to use? How should specific technologies interact with particular content, ways of reasoning, and pedagogies?*)
- How to support students' meaningful learning in the mathematical and statistical sciences through integrating ways of reasoning, content, and technologies in standards, curriculum, instruction, and assessment.
- Guidance for using the framework and on bringing changes to life in a way that coordinates across multiple systems, including policies, prerequisites, and placements.
- Guidance on how to build structures to ensure equity, inclusion, flexibility, and future adaptability that can attend to the evolving nature of the mathematical and statistical sciences.

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