CHAPTER 2

The Mathematical Education of Teachers: Traditions, Research, Current Context

This report focuses on the mathematical education of teachers, asking more mathematics departments and more mathematicians to assign high priority to teacher preparation, content-based professional development, partnerships with mathematics educators, and increased participation in the mathematics education community. To appreciate the need for changing some current priorities and practices in teacher education, it is important to understand what they are, and the traditions of school mathematics that shaped them, and still shape prospective and practicing teachers. Thus, this chapter briefly reviews traditions of teacher education and school mathematics. It is also helpful to review what is known about the mathematical knowledge needed for teaching. Thus, this chapter gives an overview of current research on teacher knowledge, and discusses it in light of the Common Core State Standards and other aspects of the current educational context.

Traditions, Beliefs, and Practices

Mathematicians' roles in teacher education. As stewards of their discipline, mathematicians have a long tradition of concerning themselves with school mathematics and its teachers. In the eighteenth century, Leonard Euler wrote an arithmetic textbook as did Augustus de Morgan a century later.[1] Felix Klein's work with high school teachers gave us the notion of "elementary mathematics from an advanced standpoint"—understanding the mathematical foundations of school mathematics. Klein was a founder of what is now the International Commission on Mathematical Instruction. Since its inception in 1908 as part of the International Mathematics Union, its presidents have included Jacques Hadamard, Marshall Stone, and other distinguished mathematicians.[2]

In the United States, as in many other countries, mathematicians' involvement in teacher preparation increased as nineteenth-century normal schools became twentieth-century colleges and universities. In 1893, the Committee of Ten, composed of presidents of Harvard and other leading universities, led the creation of influential school curriculum guidelines. Among the writers were Simon Newcomb and Henry Fine, both future presidents of the American Mathematical Society.

However, for a variety of reasons, both internal and external to the U.S. mathematics community, concern for school mathematics and its teachers did not retain

[1]These were: *Einleitung zur Rechen-Kunst* (*Introduction to the Art of Reckoning*), St Petersburg (vol. 1, 1738, vol. 2, 1740); *The Elements of Arithmetic*, London, 1830.

[2]Hodgson points out that "one could even see the ICMI as having been formed on the very assumption that university mathematicians should have an influence on school mathematics." See *The Teaching and Learning of Mathematics at University Level*, Kluwer, 2001, p. 503.

similar prominence among mathematicians during much of the twentieth century.[3] Although there have been notable counterexamples,[4] teacher education and school mathematics have often been peripheral concerns for mathematicians and mathematics departments. This situation is consistent with existing policies and practices, inside and outside of mathematics departments. Departmental support and professional development for mathematicians involved with teacher education is often sparse.[5] In the past, professional development centered on mathematics for PreK–12 teachers has been infrequent, both in general and as an activity of collegiate mathematics departments. Over the past decade, this situation has begun to change. An aim of this report is to facilitate further change.

Beliefs about mathematics and their influences on learning. As mathematicians' involvement with school mathematics decreased, the U.S. educational system expanded. Beliefs evolved—or were maintained—that shape the context of education today. Among these were students' beliefs about mathematics.

In the 1980s, education researchers began to document unmathematical beliefs among K–12 students. The statements below summarize observations of high school geometry classes where homework sets consisted of 18 to 45 problems. (Note that the first statement is counter to the first Common Core Standard for Mathematical Practice: "Make sense of problems and persevere in solving them.")

- Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.

- Ordinary students cannot expect to understand mathematics: they expect simply to memorize it and apply what they have learned mechanically and without understanding.[6]

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[3]Murray discusses the polarization of teaching and research within the U.S. mathematical community in *Women Becoming Mathematicians: Creating a Professional Identity in Post–World War II America*, MIT Press, 2000, pp. 6–10. For examples of U.S. mathematician involvement (e.g., the founding of the International Commission on the Teaching of Mathematics (later ICMI) at the International Congress of Mathematicians) and social context of its diminution, see Donoghue, "The Emergence of a Profession: Mathematics Education in the United States, 1890–1920," in *A History of School Mathematics,* vol. 1, NCTM, 2003. Changes in twentieth-century psychology research were also a factor, see Roberts, "E. H. Moore's Early Twentieth-Century Program for Reform in Mathematics Education," *American Mathematical Monthly*, 2001.

[4]*Teaching Teachers Mathematics* (Mathematical Sciences Research Institute, 2009) gives an overview of past and recent counterexamples.

[5]In 2010, Masingila et al. surveyed 1,926 U.S. higher education institutions that prepared elementary teachers. Of those who responded (43%), less than half reported giving training or support for instructors of mathematics courses for elementary teachers. However, the authors write that "there appears to be interest in training and support as a number of survey respondents contacted us to ask where they could find resources for teaching these courses." See "Who Teaches Mathematics Content Courses for Prospective Elementary Teachers in the United States? Results of a National Survey," *Journal of Mathematics Teacher Education*, 2012.

[6]Quoted from Schoenfeld, "Learning to Think Mathematically" in *Handbook for Research on Mathematics Teaching and Learning*, 1992, p. 359. Note that these beliefs may not be explicitly stated as survey or interview responses, but displayed as classroom behaviors, e.g., giving up if a problem is not quickly solved. This discussion is not meant to exclude the possibility of exceptional mathematical talent, but focuses on the idea that K–12 mathematics can be learned in its absence.

Although education researchers have identified these and other unproductive beliefs held by K–12 students, experience and other lines of research suggest that adults may hold similar beliefs about the existence of people with "math minds" or the existence of a "math gene."[7]

Recent psychological research suggests that such beliefs influence teaching and learning. This line of research has identified two distinct views. The "fixed mind-set" or "entity view of intelligence" considers cognitive abilities to be fixed from birth or unchangeable. In contrast, the "growth mind-set" or "incrementalist view" sees cognitive abilities as expandable.[8] International comparisons suggest that different views are associated with differences in achievement, and research within the U.S. has documented such associations. Students who entered seventh grade with a growth mind-set earned better grades over the next two years than peers who entered with a fixed mind-set and the same scores on mathematics tests. Classroom studies have shown that it is possible to change students' views from a fixed mind-set to a growth mind-set in ways that encourage them to persevere in learning mathematics and improve achievement test scores as well as grades.[9] Studies like these suggest that teaching practices are an important factor in reinforcing or changing students' beliefs.

Practices in teaching mathematics and their influence on learning. Unproductive beliefs about mathematics were identified in the late twentieth century, but historical research suggests that they may have been fostered by early schooling practices. Among these were pedagogical approaches. The "rule method" (memorize a rule, then practice using it) was the sole approach used in U.S. arithmetic textbooks from colonial times until the 1820s.[10] Between 1920 and 1930, pedagogy based on the work of the psychologist Edward Thorndike again emphasized memorization, e.g., memorization of arithmetic "facts" with no attempt to encourage children to notice how two facts might be related. Thus, $3 + 1 = 4$ was not connected to $1 + 3 = 4$, missing an opportunity to begin developing an understanding of the commutative law as well as the mathematical practice of seeking structure (see Appendix C). These pedagogical ideas were revived in the "back to basics" era of the 1980s and are sometimes still used, despite the existence of very different approaches that are currently used.[11]

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[7]Stevenson and Stigler documented similar beliefs among U.S. first and fifth graders, and their mothers, but found that their Japanese and Chinese counterparts focused more on effort rather than ability. See Chapter 5 of *The Learning Gap*, Simon & Schuster, 1992. See also *Data Compendium for the NAEP 1992 Mathematics Assessment for the Nation and the States*, National Center for Educational Statistics, 1993.

[8]Note that such beliefs may vary according to domain, e.g., one may believe in a "math gene," but favor continued practice in order to improve sports performance.

[9]For a brief overview of research in this area, including classroom studies, see Dweck, "Mindsets and Equitable Education," *Principal Leadership*, 2010. For a review of research and recommendations for classroom practice, see *Encouraging Girls in Math and Science* (IES Practice Guide, NCER 2007-2003), Institute of Educational Sciences, 2007, pp. 11–13.

[10]See Michalowicz & Howard, "An Analysis of Mathematics Texts from the Nineteenth Century" in *A History of School Mathematics*, vol. 1, NCTM, 2003, especially pp. 82–83.

[11]Lambdin & Walcott, "Changes through the Years: Connections between Psychological Learning Theories and the School Mathematics Curriculum," *The Learning of Mathematics*, 69th Yearbook, NCTM, 2007. For discussion of current practices, see Ma, "Three Approaches to One-Place Addition and Subtraction: Counting Strategies, Memorized Facts, and Thinking Tools," unpublished.

Other beliefs may have maintained fragile understanding of mathematics for teachers and their students, reinforcing teachers' reliance on approaches that focused on memorizing and following rules. One was the belief that elementary teachers learned all the mathematics that they needed to know during their own schooling. Such beliefs are reflected in the policies and practices noted in Chapter 1: few or no mathematics requirements for K–8 teacher preparation and certification; and infrequent professional development centered on mathematics.

In addition to identifying counterproductive beliefs about learning mathematics, mathematics education researchers have identified associated beliefs about the roles of teachers and students in mathematics classrooms:

- Doing mathematics means following the rules laid down by the teacher.
- Knowing mathematics means remembering and applying the correct rule when the teacher asks a question.
- Mathematical truth is determined when the answer is ratified by the teacher.[12]

Systematic studies of U.S. classrooms are not abundant, but their findings and those of student surveys are consistent with these descriptions of classroom expectations.[13]

Consistent with traditions for classroom behavior, videotape analyses have found far fewer occurrences of deductive reasoning in U.S. mathematics classrooms than in classrooms from countries whose students score well on international tests.[14] Moreover, studies of U.S. textbooks and curriculum documents suggest that they have often been constructed in ways that do not readily afford deductive reasoning. Such curriculum studies note imprecise, nonexistent, or contradictory definitions, or more global issues such as repetition of topics, suggesting disconnected treatments of topics with similar underlying structures (e.g., base-ten notation for whole numbers and for decimals).[15]

Summary. These traditions in U.S. school mathematics suggest that undergraduates (including prospective teachers) who have been educated in the U.S. may have well-established beliefs about mathematics and expectations for mathematics instruction that are antithetical to those of their mathematician instructors. As stated in MET I:

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[12]This is a slight reformulation of Lampert, 1990 as quoted by Schoenfeld, "Learning to Think Mathematically" in *Handbook for Research on Mathematics Teaching and Learning,* 1992, p. 359. The surrounding text discusses research on school experiences that shape such beliefs.

[13]For example, see Hiebert et al.'s study of eighth grade classrooms, *Teaching Mathematics in Seven Countries: Results from the TIMSS 1999 Video Study*, U.S. Department of Education, 2003.

[14]See analyses of data from the TIMSS video studies of 1999 (Hiebert et al., pp. 73–75) and of 1995 (Manaster, *American Mathematical Monthly*, 1998).

[15]Schmidt and Houang analyzed the content and sequencing of topics in grades 1–8 in the U.S. and other countries. See "Lack of Focus in the Mathematics Curriculum," in *Lessons Learned*, Brookings Institution Press, 2007, p. 66. Examples of treatments of fractions and negative numbers that do not afford deductive reasoning are given by Wu in "Phoenix Rising," *American Educator*, 2011.

> For many prospective teachers, learning mathematics has meant *only* learning its procedures and, they may, in fact, have been rewarded with high grades in mathematics for their fluency in using procedures. (emphasis added)

The traditions and findings described here suggest that doing mathematics in ways consistent with mathematical practice is likely to be a new, and perhaps, alien experience for many teachers. However, such experiences are necessary for teachers if their students are to achieve the Common Core State Standards for Mathematical Practice.

Although this situation may look grim, it is not intractable. Collaborations between mathematicians and mathematics educators in teacher education have made remarkable progress in developing ways to address teachers' unmathematical beliefs and practices as well as gaps in their mathematical knowledge.[16] As evidenced by outcomes from the Math Science Partnerships and research on professional development, teachers can acquire mathematical practices from carefully designed experiences of doing mathematics.[17] This suggests that doing mathematics in ways consistent with the Common Core State Standards for Mathematical Practice is an important element in the mathematical education of teachers.

Teacher Effectiveness and Mathematical Knowledge

"Teacher effectiveness" is generally construed as the effect that a teacher has on her or his students' learning. Research on teacher effectiveness often examines relationships between teacher knowledge and student achievement. In these studies, students' achievement is generally measured by standardized tests,[18] but their teachers' knowledge has been measured in quite different ways.

Mathematics courses and certification. For at least 50 years, studies of teacher effectiveness have often focused on teacher preparation, and mainly on high school and middle grades teachers. Certification status has been a popular measure. The existing evidence suggests that certification in mathematics is desirable for high school and middle grades teachers. Another measure has been the number and type of mathematics courses taken. In general, studies of high school and middle grades teachers report that more mathematics courses are associated with better performance by their students. However, these effects are small, sometimes inconsistent, and do not indicate the type of knowledge used in teaching.[19] Moreover,

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[16]For example, middle grades and high school teachers who participated in an MSP based on an immersion approach (involving intensive sessions of doing mathematics) reported changes in beliefs that affected their teaching, e.g., communicating that it is "OK" to struggle. See *Focus on Mathematics Summative Evaluation Report 2009*, p. 73. Gains in student test scores are shown on p. 93 (high school) and p. 96 (middle grades).

[17]For a snapshot from one such collaboration, see *Teaching Teachers Mathematics*, Mathematical Sciences Research Institute, 2009, p. 34; for descriptions of three Math Science Partnerships, see pp. 32–41.

[18]Test quality can be a major limitation for this measure. An analysis of state mathematics tests found low levels of cognitive demand, e.g., questions that asked for recall or performance of simple algorithms, rather than complex reasoning over an extended period. See Hyde et al., "Gender Similarities Characterize Math Performance," *Science,* 2008, pp. 494–495.

[19]See *Preparing Teachers: Building Evidence for Sound Policy*, National Research Council, 2010, p. 112. See also, Telese, "Middle School Mathematics Teachers' Professional Development

certification or undergraduate course-taking are quite imprecise measures, due to variability in certification requirements and undergraduate instruction.

Mathematical knowledge for teaching. A different line of research has begun to offer evidence that particular forms of mathematical knowledge are important in teaching. In the 1980s, scholars began to investigate "knowledge for teaching," criticizing earlier research on effectiveness for ignoring the subject matter and its transformation into the content of instruction.[20] Initially, this line of research analyzed the actions of teachers in classrooms or outcomes of interviews with teachers, rather than survey data and test scores. The focus was on identifying kinds of knowledge relevant for *teaching* mathematics, rather than mathematical knowledge in general. For example, prospective teachers were asked to respond to classroom scenarios, such as a question about why division by 0 is undefined. Responses indicated that even mathematics majors were not always able to answer in a satisfactory way.[21]

As noted in MET I, such interviews with teachers awakened many mathematicians to the special nature of mathematics for teaching and its implications for the education of teachers. Since that time, this line of research has continued toward developing tests of mathematical knowledge for teaching. Third-grade teachers' scores on one such test (Learning Mathematics for Teaching) were better predictors of their students' achievement than measures such as average time spent in mathematics instruction, years of experience, and certification status.[22]

Curriculum-specific professional development. A second line of recent research has focused on studying relationships between teachers' professional development experiences and their students' performance on mathematics tests. A 1998 study of professional development in California found that attending workshops that were mathematics- and curriculum-specific (e.g., as opposed to learning to use manipulatives or to improve classroom management) was associated with better student performance on mathematics tests.[23] A 2009 meta-analysis of professional development studies found that those in which teachers focused, for a sustained period, on examining mathematics underlying the curriculum and how to teach it were associated with improved student achievement.[24] Similarly, a project in which a research-based "toolkit" on fractions was supplied to treatment groups of U.S. elementary teachers to use in lesson study found that groups who used the toolkit

and Student Achievement," *Journal of Educational Research*, 2012. Telese's measure of student achievement was the Grade 8 National Assessment of Educational Progress, which includes items with a high level of cognitive demand. It found number of mathematics courses to be a strong predictor, but like many such studies, it did not have an experimental or quasi-experimental design.

[20]Shulman, "Those Who Understand: Knowledge Growth in Teaching," *Educational Researcher*, 1986.

[21]On average, the prospective secondary teachers had taken over 9 college-level mathematics courses. Ball, "Prospective Elementary and Secondary Teachers' Understanding of Division," *Journal for Research in Mathematics Education*, 1990.

[22]Hill et al., "Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement," *American Educational Research Journal*, 2005.

[23]Cohen & Hill, "Instructional Policy and Classroom Performance: The Mathematics Reform in California," *Teachers College Record*, 2000.

[24]Blank & Atlas, *Effects of Teacher Professional Development on Gains in Student Achievement*, Council of Chief State School Officers, 2009.

were associated with significantly greater student achievement than those of control groups.[25]

Teaching–learning paths. A third line of research on teacher effectiveness focuses on learning trajectories—sequences of student behaviors indicating different levels of thinking with instructional tasks that lead to development of a mathematical ability. Related ways to focus instruction are described as teaching–learning paths, "learning lines," and learning progressions. These notions, together with examples of paths from U.S. research and curriculum materials from other countries, informed the development of the CCSS.

An example from China may help to illustrate the general nature of these U.S. notions. Chinese teachers describe a sequence of problems together with concepts and skills that lead students to be able to compute whole-number subtraction problems with regrouping (e.g., 104 – 68), and to understand the rationale for their computations. Each part of the sequence involves a new kind of problem, a new idea, and a new skill.

Minuends between 10 and 20, e.g., $15 - 7$, $16 - 8$	New concept and skill of decomposing a ten.
Minuends between 19 and 100, e.g., $53 - 25$, $72 - 48$	New concept and skill of splitting off a ten, followed by decomposing a ten.
Minuends with three or more digits.	New concept and skill of successive decomposition.[26]

In the U.S., randomized studies of preschool classrooms have shown large student gains for a curriculum based on learning trajectories that included sustained and specific professional development for teachers.[27] Studies of elementary grades have focused on assessment tasks, rather than entire curricula. But, like the curriculum for the preschool classrooms, these tasks outline a learning path that goes step by step, helping students incrementally increase their understanding, as they move toward a mathematical goal. They also create a teaching path, helping teachers perceive the elements of a given concept or skill, and mathematical stepping-stones in their development.[28]

Large-scale studies that examine connections between student achievement in earlier and later grades suggest that improved mathematics instruction in preschool and elementary grades has a large payoff in later achievement, not only for mathematics in later grades (including high school), but for reading.[29] Such studies

[25]Perry & Lewis, *Improving the Mathematical Content Base of Lesson Study: Summary of Results*, 2011.

[26]Example from Ma, *Knowing and Teaching Elementary Mathematics*, Erlbaum, 1999, p. 15. Similar examples occur in other East Asian countries. Lewis et al. describe how Japanese teacher's manuals may support teachers' perceptions of paths in "Using Japanese Curriculum Materials to Support Lesson Study Outside Japan: Toward Coherent Curriculum," *Educational Studies in Japan: International Yearbook*, 2011.

[27]Sarama & Clements, *Early Childhood Mathematics Education Research*, Routledge, 2009, pp. 352–363.

[28]See special issue on learning trajectories, *Mathematical Thinking and Learning*, 2004.

[29]See Duncan et al., "School Readiness and Later Achievement," *Developmental Psychology*, 2007; Claessens et al., "Kindergarten Skills and Fifth-grade Achievement: Evidence from the ECLS-K," *Economics of Education Review*, 2009; Siegler et al., "Early Predictors of High

reiterate the importance of mathematics in preparation and professional development for early childhood and elementary teachers.

Summary. Studies of teacher effectiveness suggest that mathematics course-taking and certification are desirable for middle grades and high school teachers, but are inconclusive about the nature of the mathematical knowledge that teachers need. However, the existing evidence suggests that teacher preparation and professional development should be tailored to the work of teaching. The National Research Council study *Preparing Teachers* concludes:

> Current research and professional consensus correspond in suggesting that all mathematics teachers ... rely on: mathematical knowledge for teaching, that is, knowledge not just of the content they are responsible for teaching, but also of the broader mathematical context for that knowledge and the connections between the material they teach and other important mathematics content.[30]

Within the U.S., such knowledge is not currently well developed in the profession of mathematics teaching. Mathematicians are among those necessary for its development.

For PreK–8 teachers, adequate preparation includes more mathematics than often thought. Moreover, studies connecting teachers' understanding of teaching–learning paths and student achievement show how the organization of curriculum together with attention to teacher knowledge can work together to improve students' learning. A necessary first step for teachers is to understand the mathematics in these paths,[31] thus mathematicians' participation in their education is essential.

Current Context

Since MET I was published in 2001, there have been significant changes in teacher education: outside mathematics departments with respect to the teaching workforce and educational policy; within mathematics departments with respect to courses for teachers and faculty involvement in K–12 education.

Demographic changes have occurred for the teaching workforce as a whole. Analyses of nationally representative survey data find that between 1988 and 2008, the age distribution for teachers shifted from a unimodal distribution with a peak at age 41 to a bimodal distribution with peaks at ages 26 and 55. Some of these changes appear to be due to increases in the numbers of teachers for special education, elementary enrichment, science, and mathematics.[32]

In 2000, approximately 22% of secondary schools reported serious difficulties in filling teaching positions for mathematics. This dropped to about 18% in 2008. Such staffing difficulties tended to occur at high-poverty, high-minority public schools in both urban and rural areas. Over half of the teachers who left these

School Mathematics Achievement," *Psychological Science*, 2012. These studies examined large longitudinal data sets from the U.S. and other countries.

[30] *Preparing Teachers: Building Sound Evidence for Sound Policy*, National Research Council, 2010, pp. 114–115.

[31] This is made explicit for early childhood educators in *Mathematics Learning in Early Childhood*, National Research Council, 2009, pp. 3–4.

[32] Ingersoll & Merrill, "Who's Teaching Our Children?," *Educational Leadership*, 2010.

schools reported dissatisfaction or the intention to pursue another or better job.[33] Analysis of 2004 and 2005 data found differences in rates of teacher attrition at schools in the same district. Also, mathematics teachers who moved from one teaching job to another were most likely to move to schools with similar enrollments of poor and minority students. This suggests that attrition is not simply a matter of school demographics, but of school organization. An organizational factor of particular relevance to the MET II report is provision of content-based professional development. Mathematics teachers who received it and perceived it as useful had substantially lower odds of turnover.[34]

About 40% of practicing teachers have been prepared via an alternative pathway, that is, outside of a traditional teacher education program. Like standard programs, these alternative pathways vary widely.[35] Such differences can affect the rest of a teacher's career. Analyses of recent survey data find that in the first year of teaching, teachers with a mathematics baccalaureate, but little or no pedagogical preparation, left teaching at twice the rate of those with the same degree, but more comprehensive pedagogical preparation.[36]

A new accreditation organization with significantly different standards for teacher preparation is coming into existence. The Council for the Accreditation of Educator Preparation (CAEP) will require that the mathematical preparation of teachers address the CCSS.[37] In the past, accreditation requirements for mathematics have often been met by reporting results on tests such as the Praxis or course grades for appropriate courses, although other options were available. The new requirements for mathematics courses will be similar in nature to the current, more detailed, accreditation requirements for methods courses. The standards for these courses have changed to include standards for mathematical practice and to reflect the content of the CCSS.

Requirements for professional development have been changing. By 2008, all 50 states had specified professional development requirements for teachers. The majority of these require 6 semester-hours of professional development over approximately 5 years. Twenty-four of these states have a policy specifying that professional development be aligned with state content standards.[38]

More mathematics departments have designed courses especially for K–8 teachers or have designated special sections of courses for these teachers.[39] In some

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[33] Ingersoll & Perda, "Is the Supply of Mathematics and Science Teachers Sufficient?," *American Educational Research Journal*, 2010.

[34] Ingersoll & May, "The Magnitude, Destinations, and Determinants of Mathematics and Science Teacher Turnover," Consortium for Policy Research in Education, 2010, pp. 44, 46.

[35] *Preparing Teachers: Building Sound Evidence for Sound Policy*, National Research Council, 2010, pp. 34–39.

[36] Ingersoll & Merrill, "Retaining Teachers: How Preparation Matters," *Educational Leadership*, 2012. See also Darling-Hammond, *Solving the Dilemmas of Teacher Supply, Demand, and Standards*, National Commission on Teaching and America's Future, 2000, pp. 17–19; *Tenth Anniversary Report*, UTeach, 2010, p. 16.

[37] CAEP was formed by the merger of the National Council for the Accreditation of Teacher Education (NCATE) and the Teacher Education Accreditation Council (TEAC). Two of the MET II writers are engaged in the development of the CAEP standards.

[38] *Key State Education Policies on PK–12 Education: 2008*, Council of Chief State School Officers, p. 22.

[39] CBMS 2005 Survey, Table SP.3.

departments, policies for faculty have changed to facilitate their involvement in activities for increasing K–12 student achievement.[40]

Collectively, the mathematics community now has substantial experience in developing partnerships that allow teachers to achieve the goals for teacher preparation and professional development described in this report and others.[41] Collaboration with others in mathematics education has allowed mathematicians to have a major impact on professional development within states.[42] Partnerships that began in the 1990s have expanded in scope or have been duplicated at multiple locations. Mathematicians have expanded their involvement in mathematics education, forming partnerships with mathematics education researchers, education officials, and teachers in new kinds of programs. Through these experiences, concerned mathematicians gained greater expertise and awareness about the challenges to improving mathematical learning in the schools, and within states. More information about these and other relevant efforts is on the web page associated with this report. Because the CCSS have been adopted by most states, many of these projects will be able to share details and specifics about students' and teachers' learning of mathematics in ways that can be readily transported across state lines.

This is a time of great opportunity for mathematics education in the United States. Lines of communication have been opened among policy-makers, mathematicians, and mathematics educators, and changed educational policies provide the potential for educational improvement. Mathematicians have an essential role to play in fulfilling this potential in teacher education, curriculum, and assessment.

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[40] *National Impact Report: Math and Science Partnership Program*, National Science Foundation, 2010, p. 15.

[41] In addition to the forthcoming CAEP standards, note the 2012 report *Supporting Implementation of the Common Core State Standards for Mathematics: Recommendations for Professional Development*, Friday Institute for Educational Innovation at the North Carolina State University College of Education.

[42] For an overview of MSP outcomes, including increases in student achievement, see *National Impact Report: Math and Science Partnership Program*, National Science Foundation, 2010, pp. 6, 10–12.