CHAPTER 4

Elementary Teachers

What mathematics should future elementary teachers study to prepare for their careers? What mathematics coursework and programs will prepare elementary teachers for teaching mathematics? What sorts of professional development experiences will develop and sustain high quality mathematics teaching in elementary school? How can mathematicians make valuable contributions to these endeavors? These questions are the topics of this chapter. Coursework in mathematical pedagogy is assumed to be part of a preparation program, but is not discussed in detail.

In this chapter, the term "elementary teacher" is defined as a teacher who teaches mathematics at the K–5 level.[1]

A major advance in teacher education is the realization that teachers should study the mathematics they teach in depth, and from the perspective of a teacher. There is widespread agreement among mathematics education researchers and mathematicians that it is not enough for teachers to rely on their past experiences as learners of mathematics.[2] It is also not enough for teachers just to study mathematics that is more advanced than the mathematics they will teach. Importantly, mathematics courses and professional development for elementary teachers should not only aim to remedy weaknesses in mathematical knowledge, but also help teachers develop a deeper and more comprehensive view and understanding of the mathematics they will or already do teach.

Thus, this report recommends that before beginning to teach, an elementary teacher should study in depth, and from a teacher's perspective, the vast majority of K–5 mathematics, its connections to prekindergarten mathematics, and its connections to grades 6–8 mathematics. By itself, this expectation is not sufficient to guarantee high quality teaching. In particular, teachers will also need courses in

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Note that the MET II web resources at `www.cbmsweb.org` give URLs for the CCSS, the Progressions for the CCSS, and other relevant information.

[1]As noted in Chapter 3, "Although elementary certification in most states is still a K–6 and, in some states, a K–8 certification, state education departments and accreditation associations are urged to require all grades 5–8 teachers of mathematics to satisfy the 24-hour requirement recommended by this report." Chapters 4 and 5 allow for a period of transition.

[2]For example, "It is widely assumed—some would claim common sense—that teachers must know the mathematical content they teach" (*Foundations for Success: Reports of Task Groups of the National Mathematics Advisory Panel*, 2008, p. 5-6). "Aspiring elementary teachers must begin to acquire a deep conceptual knowledge of the mathematics that they will one day need to teach, moving well beyond mere procedural understanding" (*No Common Denominator*, 2008, National Council on Teacher Quality). "Mathematics courses for future teachers should develop 'deep understanding' of mathematics, particularly of the mathematics taught in schools at their chosen grade level" (Curriculum Foundations Project, 2001, Mathematical Association of America). See also *Preparing Teachers: Building Sound Evidence for Sound Policy*, 2010, National Research Council, p. 123.

mathematical pedagogy. However, there is no substitute: a strong understanding of the mathematics a teacher will teach is necessary for good teaching. Every elementary student deserves a teacher who knows, very well, the mathematics that the student is to learn. As reasonable as this expectation may seem, it is not routinely achieved.[3]

With the advent of the Common Core State Standards for Mathematics (CCSS), there is now a succinct description of the mathematics to be taught and learned at the elementary school level in the United States. The CCSS describe not only the specific mathematical skills and understandings that students are to acquire but also the kinds of mathematical practice that students are to develop.

Several points about the CCSS Standards for Mathematical Practice bear emphasizing. First, although those standards were written for K–12 students, they apply to all who do mathematics, including elementary teachers. Second, the features of mathematical practice described in these standards are not intended as separate from mathematical content. Teachers should acquire the types of mathematical expertise described in these standards as they learn mathematics. And finally, *engaging in mathematical practice takes time and opportunity*, so that coursework and professional development for teachers must be planned with that in mind. Time and opportunity to think about, discuss, and explain mathematical ideas are essential for learning to treat mathematics as a sense-making enterprise.

Readers who are new to the preparation and professional development of elementary teachers may find some of the ideas, examples, and terms (e.g., "unit rate," "tape diagram") presented in this chapter unfamiliar or unusual. Interested readers, and those who will teach mathematics courses and provide professional development for teachers, should consult additional sources for definitions and examples, including the CCSS and the Progressions for the CCSS (see the web resources associated with this report). Materials that have been carefully designed for courses and professional development opportunities for teachers exist and are a sensible starting point for those who will begin teaching such courses and providing professional development.

What kinds of problems might prospective or practicing elementary teachers work on in coursework or in professional development experiences? What kinds of mathematical discussions, explanations, and thinking might they engage in? The first section of this chapter gives very brief sketches of how the mathematics might be treated in coursework or professional development for teachers, showing its difference from the content of courses often taken by teachers, e.g., college algebra.

The second section of this chapter suggests how this mathematics can be organized in courses, programs, or seminars for prospective or practicing elementary teachers. In addition, this section describes other types of professional development for teachers that afford opportunities for mathematicians to participate in the broader mathematics education community. The final sections of the chapter

[3]An international comparison of prospective elementary teachers found that 48% of the U.S. teachers did not score above "Anchor point 2." Teachers with this score often had trouble reasoning about factors, multiples, and percentages. See Tatto & Senk, "The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics," *Journal of Mathematics Teacher Education*, 2011, pp. 129–130. *Preparing Teachers* discusses concern about the adequacy of current teacher preparation in mathematics, especially for elementary teachers. See Chapter 6, especially p. 124.

discuss the preparation and professional development of elementary mathematics specialists, early childhood teachers, and teachers of special populations.

Essential Grades K–5 Ideas for Teachers

This section uses the CCSS as a framework for outlining the mathematical ideas that elementary teachers, both prospective and practicing, should study and know. The CCSS standards for mathematical content are organized into clusters of related standards and the clusters are organized into mathematical domains, which span multiple grade levels (see Appendix B). Brief descriptions of how the mathematics of each domain progresses across grade levels and is connected within or across grades to standards in other domains appear in the Progressions for the CCSS (see the web resources associated with this report).

Because elementary teachers prepare their students for the middle grades, courses and seminars for elementary teachers should also attend to how the mathematical ideas of the elementary grades build to those at the middle grades, and should highlight connections between topics at the elementary and middle levels. Thus, courses and professional development will need to devote time to ideas within the middle grades domains of Ratio and Proportional Relationships, The Number System, Expressions and Equations, and Statistics and Probability (see Chapter 5).

This section lists essential ideas of each K–5 domain and important connections to prior or later grades that teachers need to know well. These listings are not intended as comprehensive; and instructors are encouraged to refer to the CCSS, related progressions, and other references given in the web resources for further details and discussion.

For each domain, the list of essential ideas is followed by a list of related activities that could be used in teacher preparation or professional development.

A given activity may provide opportunities to demonstrate or develop various kinds of expertise described by one or more of the CCSS standards for mathematical practice. These are indicated by the number and heading of the associated standard. For example, "MP 1 Make sense of problems and persevere in solving them" indicates expertise connected with the first Standard for Mathematical Practice might be used. (The full text for all eight Standards for Mathematical Practice appears as Appendix C of this report.) Note that although a particular activity might provide opportunities to use or increase expertise, instructors should expect to foster engagement in these opportunities. Also, instructors might periodically remind teachers to review and reflect on the Standards for Mathematical Practice so that they become more familiar with the types of expertise described by these standards in the context of elementary mathematics.

Counting and Cardinality (Kindergarten)

- The intricacy of learning to count, including the distinction between counting as a list of numbers in order and counting to determine a number of objects.

Illustrative activity:

Examine counting errors that young children typically make and study the learning path of counting.[4] (This includes connections to prekindergarten mathematics.)

MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.

Operations and Algebraic Thinking (Kindergarten–Grade 5)

- The different types of problems solved by addition, subtraction, multiplication, and division, and meanings of the operations illustrated by these problem types.[5]
- Teaching–learning paths for single-digit addition and associated subtraction and single-digit multiplication and associated division, including the use of properties of operations (i.e., the field axioms).
- Recognizing the foundations of algebra in elementary mathematics, including understanding the equal sign as meaning "the same amount as" rather than a "calculate the answer" symbol.

Illustrative activities:

(1) Write equations for addition and subtraction problems of different types and determine which cases have a "situation equation" (an equation that fits naturally with the wording of the problem) that is different from a "solution equation" (an equation that is especially helpful for solving the problem).

MP 2 Reason abstractly and quantitatively.

(2) Recognize that commutativity for multiplication is not obvious and use arrays to explain why multiplication is commutative.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 5 Use appropriate tools strategically.

(3) Explain why we can't divide by 0.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.

(4) Explore and discuss the different ways remainders can be interpreted when solving division problems.

MP 4 Model with mathematics.
MP 6 Attend to precision.

(5) Explain how to solve equations such as $283 + 19 = x + 18$ by "thinking relationally" (e.g., by recognizing that because 19 is 1 more than 18, x should be 1 more than 283 to make both sides equal) rather than by applying standard algebraic methods.

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[4] See the National Research Council report *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* and the Counting and Cardinality Progression.

[5] See CCSS, pp. 88–89; or the Operations and Algebraic Thinking Progression for details and examples of situation and solution equations.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Number and Operations in Base Ten (Kindergarten–Grade 5)

- How the base-ten place value system relies on repeated bundling in groups of ten and how to use objects, drawings, layered place value cards, and numerical expressions to help reveal base-ten structure. Developing progressively sophisticated understandings[6] of base-ten structure as indicated by these expressions:

$$\begin{aligned} 357 &= 300 + 50 + 7 \\ &= 3 \times 100 + 5 \times 10 + 7 \times 1 \\ &= 3 \times (10 \times 10) + 5 \times 10 + 7 \times 1 \\ &= 3 \times 10^2 + 5 \times 10^1 + 7 \times 10^0. \end{aligned}$$

- How efficient base-ten computation methods for addition, subtraction, multiplication, and division rely on decomposing numbers represented in base ten according to the base-ten units represented by their digits and applying (often informally) properties of operations, including the commutative and associative properties of addition and the distributive property, to decompose the calculation into parts. How to use math drawings or manipulative materials to reveal, discuss, and explain the rationale behind computation methods.
- Extending the base-ten system to decimals and viewing decimals as address systems on number lines. Explaining the rationales for decimal computation methods. (This includes connections to grades 6–8 mathematics.)

Illustrative activities:

(1) Make simple base-ten drawings to calculate $342 - 178$ and identify correspondences with numerical written methods. Compare advantages and disadvantages to each numerical written variation.

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.

(2) Examine hypothetical or actual student calculation methods and decide if the methods are valid or not. For example, recognize that if a student calculates 23×45 by calculating 20×40 and 3×5 and adding the two results, the method is not legitimate but can be modified to become correct by adding the two missing products that arise from applying the distributive property, which can also be seen in an area or array model.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.

[6] For examples of how teachers may construe the base-ten system, see Thanheiser, "Pre-service Elementary School Teachers' Conceptions of Multidigit Whole Numbers," *Journal for Research in Mathematics Education*, 2009.

(3) Explain how to use properties of operations to make some calculations such as 98×15 or 24×25 easy to carry out mentally and write strings of equations, such as $24 \times 25 = (6 \times 4) \times 25 = 6 \times (4 \times 25) = 6 \times 100 = 600$, to show how properties of operations support the "mental math."

MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.

Number and Operations—Fractions (Grades 3–5)

- Understanding fractions as numbers which can be represented with lengths and on number lines. Using the CCSS development of fractions to define fractions a/b as a parts, each of size $1/b$. Attending closely to the whole (referent unit) while solving problems and explaining solutions.
- Recognizing that addition, subtraction, multiplication, and division problem types and associated meanings for the operations (e.g., CCSS, pp. 88–89) extend from whole numbers to fractions.
- Explaining the rationale behind equivalent fractions and procedures for adding, subtracting, multiplying, and dividing fractions. (This includes connections to grades 6–8 mathematics.)
- Understanding the connection between fractions and division, $a/b = a \div b$, and how fractions, ratios, and rates are connected via unit rates. (This includes connections to grades 6–8 mathematics. See the Ratio and Proportion Progression for a discussion of unit rate.)

Illustrative activities:

(1) Use drawings and reasoning to solve problems and explain solutions. For example:

> One serving of rice is $1/2$ cup. You ate $2/3$ of a cup of rice. How many servings did you eat?

Examine and critique reasoning, such as:

> A student said that $2/3$ of a cup of rice is 1 serving plus another $1/6$. Is that correct? [It is 1 serving plus another $1/6$ of a cup of rice, but the $1/6$ of a cup of rice is $1/3$ of a serving. That is because $1/2 = 3/6$. The $1/6$ of a cup of rice is one of the 3 sixths of a cup that make a $1/2$ cup serving.]

MP 1 Make sense of problems and persevere in solving them.
MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 5 Use appropriate tools strategically.

(2) Give rationales underlying methods for comparing fractions, including comparing fractions with common denominators or common numerators and explain how to compare fractions by relating them to benchmarks such as $1/2$ or 1. For example, use reasoning to compare $73/74$ and $85/86$.

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.

(3) Explain how it can happen that the multiplication of fractions can produce a product that is smaller that its factors and division of fractions can produce a quotient that is larger than divisor and dividend.

MP 2 Reason abstractly and quantitatively.

(4) Calculate percentages mentally and write equations to show the algebra behind the mental methods, such as calculating 45% of 120 by taking half of 120, which is 60, then taking away 10% of that, leaving 54.

$$\begin{aligned}\text{Possible equations: } 45\% \cdot 120 &= (50\% - 5\%)120 \\ &= 50\% \cdot 120 - 5\% \cdot 120 \\ &= 60 - (10\% \cdot \tfrac{1}{2})120 \\ &= 60 - 10\%(\tfrac{1}{2} \cdot 120) \\ &= 60 - 10\% \cdot 60 \\ &= 60 - 6 = 54.\end{aligned}$$

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.

Measurement and Data (Kindergarten–Grade 5)

- The general principles of measurement, the process of iterations, and the central role of units: that measurement requires a choice of measureable attribute, that measurement is comparison with a unit and how the size of a unit affects measurements, and the iteration, additivity, and invariance used in determining measurements.
- How the number line connects measurement with number through length (see the Geometric Measurement Progression).
- Understanding what area and volume are and giving rationales for area and volume formulas that can be obtained by finitely many compositions and decompositions of unit squares or unit cubes, including formulas for the areas of rectangles, triangles, and parallelograms, and volumes of rectangular prisms. (This includes connections to grades 6–8 geometry, see the Geometric Measurement Progression.)
- Using data displays to ask and answer questions about data. Understanding measures used to summarize data, including the mean, median, interquartile range, and mean absolute deviation, and using these measures to compare data sets. (This includes connections to grades 6–8 statistics, see the Measurement Data Progression.)

Illustrative activities:

(1) Explore the distinction and relationship between perimeter and area, such as by fixing a perimeter and finding the range of areas possible or by fixing an area and finding the range of perimeters possible.

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 8 Look for and express regularity in repeated reasoning.

(2) Investigate whether the area of a parallelogram is determined by the lengths of its sides. Given side lengths, which parallelogram has the largest area? Explain how to derive the formula for the area of a parallelogram, including for "very oblique" cases, by decomposing and recomposing parallelograms and relating their areas to those of rectangles.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

(3) Examine the distinction between categorical and numerical data and reason about data displays. For example:

> Given a bar graph displaying categorical data, could we use the mean of the frequencies of the categories to summarize the data? [No, this is not likely to be useful.]
>
> Given a dot plot displaying numerical data, can we calculate the mean by adding the frequencies and dividing by the number of dots? [No, this is like the previous error.]

MP 3 Construct viable arguments and critique the reasoning of others.
MP 5 Use appropriate tools strategically.

(4) Use reasoning about proportional relationships to argue informally from a sample to a population. For example:

> If 10 tiles were chosen randomly from a bin of 200 tiles (e.g., by selecting the tiles while blindfolded), and if 7 of the tiles were yellow, then about how many yellow tiles should there be in the bin? Imagine repeatedly taking out 10 tiles until a total of 200 tiles is reached. What does this experiment suggest? Then investigate the behavior of sample proportion by taking random samples of 10 from a bin of 200 tiles, 140 of which are yellow (replacing the 10 tiles each time). Plot the fraction of yellow tiles on a dot plot or line plot and discuss the plot. How might the plot be different if the sample size was 5? 20? Try these different sample sizes.

MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.

Geometry (Kindergarten–Grade 5)

- Understanding geometric concepts of angle, parallel, and perpendicular, and using them in describing and defining shapes; describing and reasoning about spatial locations (including the coordinate plane).
- Classifying shapes into categories and reasoning to explain relationships among the categories.
- Reason about proportional relationships in scaling shapes up and down. (This is a connection to grades 6–8 geometry.)

Illustrative activities:

(1) Explore how collections of attributes are related to categories of shapes. Sometimes, removing one attribute from a collection of attributes does not change the set of shapes the attributes apply to and sometimes it does.

MP 7 Look for and make use of structure.

(2) Reason about scaling in several ways: If an 18-inch by 72-inch rectangular banner is scaled down so that the 18-inch side becomes 6 inches, then what should the length of the adjacent sides become? Explain how to reason by:

> Comparing the 18-inch and 6-inch sides. [The 18-inch side is 3 times the length of the 6-inch side, so the same relationship applies with the 72-inch side and the unknown side length.]
>
> Comparing the 18-inch and 72-inch sides. [The 72-inch side is 4 times the length of the 18-inch side, so the unknown side length is also 4 times the length of the 6-inch side.]

MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

The Preparation and Professional Development of Elementary Teachers

The mathematics of elementary school is full of deep and interesting ideas, which can be studied repeatedly, with increasing depth and attention to detail and nuance. Therefore, although prospective teachers will undertake an initial study of elementary mathematics from a teacher's perspective in their preparation program, practicing teachers will benefit from delving more deeply into the very same topics. Perhaps surprisingly, mathematics courses that explore elementary school mathematics in depth can be genuinely college-level intellectual experiences, which can be interesting for instructors to teach and for teachers to take.[7]

Programs for Prospective Teachers

Programs designed to prepare elementary teachers should include 12 semester-hours focused on a careful study of mathematics associated with the CCSS (K–5 and related aspects of 6–8 domains) from a teacher's perspective. This includes, but is not limited to studying all the essential ideas described in the previous section and their connections with the essential ideas of grades 6–8 described in Chapter 5. It also includes some attention to methods of instruction. Number and operations, treated algebraically with attention to properties of operations, should occupy about 6 of those hours, with the remaining 6 hours devoted to additional ideas of algebra (e.g., expressions, equations, sequences, proportional relationships, and linear relationships), and to measurement and data, and to geometry.

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[7]Beckmann, "The Community of Math Teachers, from Elementary School to Graduate School," *Notices of the American Mathematical Society*, 2011.

When possible, program designers should consider courses that blend the study of content and methods. Prospective teachers who have a limited mathematical background will need additional coursework in mathematics.

It bears emphasizing that familiar mathematics courses such as college algebra, mathematical modeling, liberal arts mathematics, and even calculus or higher level courses *are not an appropriate substitute for the study of mathematics for elementary teachers*, although they might make reasonable additions.[8] Also, it is unlikely that knowledge of elementary mathematics needed for teaching can be acquired through experience in other professions, even mathematically demanding ones.

Professional development for practicing teachers

Once they begin teaching, elementary teachers need continuing opportunities to deepen and strengthen their mathematical knowledge for teaching, particularly as they engage with students and develop better understanding of their thinking.

Professional development may take a variety of forms. A group of teachers might work together in a professional learning community, and they might choose to focus deeply on one topic for a period of time. For example, the teachers at the same grade level in several schools might spend a term studying fractions in the CCSS, the grade 3–5 Fractions Progression and other curriculum documents, followed by designing, teaching, and analyzing lessons on fraction multiplication using a lesson study format.[9] Or a group of teachers who teach several grade levels at one school might meet regularly to study how related topics progress across grade levels. A group of teachers might watch demonstration lessons taught by a mathematics specialist and then meet to discuss the lessons, plan additional lessons, and study the mathematics of the lessons.[10] Teachers might also complete mathematics courses specifically designed as part of a graduate program for elementary teachers. Professional development can take place at school, either during or after school hours, or on college campuses after school hours or during the summer. However it is organized, as discussed in Chapter 2, the best professional development is ongoing, directly relevant to the work of teaching mathematics, and focused on mathematical ideas.

Regardless of format, as part of a professional development program, teachers could study mathematics materials specifically designed for professional development and, if the textbook series is carefully designed, the teacher's guides for the mathematics textbooks used at their schools. Mathematics specialists or college-based mathematics educators or mathematicians might lead sessions in which they engage teachers in solving problems, thinking together, and discussing mathematical ideas. Teachers could bring student work to share and discuss with the group. Opportunities to examine how students are thinking about mathematical ideas,

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[8] For instance, a study of prospective elementary and secondary teachers found that many either did not know that division by 0 was undefined or were unable to explain why it was undefined. On average, the secondary teachers had taken over 9 college-level mathematics courses. Ball, "Prospective Elementary and Secondary Teachers' Understanding of Division," *Journal for Research in Mathematics Education*, 1990.

[9] Lesson study is a process in which teachers jointly plan, observe, analyze, and refine actual classroom lessons.

[10] See this chapter's section on mathematics specialists for more discussion about their roles in professional development for teachers.

and to learn about learning paths and tasks designed to help students progress along learning paths are especially important for elementary teachers and can lead to improved student outcomes. Together, teachers could write problems for their students that they design to get a sense of what students already know about an upcoming topic of instruction (an example of formative assessment). They could share results of assessments and, based on the outcome, plan appropriate tasks for the students. Throughout, outside experts, such as mathematicians, statisticians and mathematics educators in higher education or professionals from mathematically-intensive fields could work with the teachers to bring a fresh perspective and to help teachers go deeply into the content. A side benefit of this work to those in higher education is the opportunity to think about undergraduate mathematics teaching and the connection between college-level mathematics courses and K–12 education.

Engaging in mathematical practice. Teacher preparation and professional development can provide opportunities to *do* mathematics and to develop mathematical habits of mind. Teachers must have time, opportunity, and a nurturing environment that encourages them to make sense of problems and persevere in solving them. They should experience the enjoyment and satisfaction of working hard at solving a problem so that they realize this sort of intellectual work can be satisfying and so that they don't seek to shield their students from the struggles of learning mathematics. Teachers should have time and opportunity to reason abstractly and quantitatively, to construct viable arguments, to listen carefully to other people's reasoning, and to discuss and critique it. Some teachers may not realize that procedures and formulas of mathematics can be explained in terms of more fundamental ideas and that deductive reasoning is considered an essential part of mathematics. Teachers should have the opportunity to model with mathematics and to *mathematize* situations by focusing on the mathematical aspects of a situation and formulating them in mathematical terms. Elementary teachers should know ways to use mathematical drawings, diagrams, manipulative materials, and other tools to illuminate, discuss, and explain mathematical ideas and procedures. Teachers need practice being precise and deliberate when they discuss their reasoning, and to be on the lookout for incomplete or invalid arguments. Especially important is that teachers learn to use mathematical terminology and notation correctly. And finally, teachers need opportunities to look for and use regularity and structure by seeking to explain the phenomena they observe as they examine different solution paths for the same problem.

Use technology and other tools strategically. Since the publication of MET I, the technology available to support the teaching and learning of mathematics has changed dramatically. These tools include interactive whiteboards and tablets, mathematics-specific technology such as virtual manipulatives and "quilting" software, and an ever-expanding set of applets, apps, web sites, and multimedia materials. Thus, it is important that teacher preparation and professional development programs provide opportunities for teachers to use these tools in their own learning so that they simultaneously advance their mathematical thinking, expand the repertoire of technological tools with which they are proficient, and develop an awareness of the limitations of technology. Teachers should have experiences using technology as a computational and problem solving tool. When technology is used as a computational tool, learners use it to perform a calculation or produce a graph or table in order to use the result as input to analyze a mathematical situation. They should also learn to use technology as a problem solving tool, or to conduct an investigation by taking a deliberate mathematical action, observing the consequences, and reflecting on the mathematical implications of the consequences. Teachers must have opportunities to engage in the use of a variety of technological tools, including those designed for mathematics and for teaching mathematics, to explore and deepen their understanding of mathematics, even if these tools are not the same ones they will eventually use with children.

Technology is one of many tools available for learning and teaching mathematics. Others are traditional tools of teaching such as blackboards.[11] Some are manipulative materials such as base-ten blocks, which can be used for early work with place value and operations with whole numbers and decimals; pattern blocks, which can be used for work with fractions; tiles; and counters. Teachers need to develop the ability to critically evaluate the affordances and limitations of a given tool, both for their own learning and to support the learning of their students. In mathematics courses for teachers, instructors should model successful ways of using such tools, and provide opportunities to discuss mathematical issues that arise in their use.

Challenges in the Education of Elementary Teachers

Prospective elementary school teachers frequently come to their teacher preparation programs with their own views about what it means to know and do mathematics and how it is learned. They sometimes feel insecure about their own mathematical knowledge while believing that learning to teach is a matter of learning to explain procedures clearly and assembling a toolkit of tasks and activities to use with children. As discussed in Chapter 2, some teachers may have a "fixed mind-set" about learning rather than a "growth mind-set" and may not recognize that everyone can improve their capacity to learn and understand mathematics. Instructors need to recognize that the messages of their courses and professional development opportunities may be filtered through such views. Some prospective teachers, although they may not like mathematics or feel confident in their ability to do it, do not think they need to learn more mathematics. In particular, they do not think there is anything else for them to learn about the content of elementary school mathematics. Similarly, prospective and practicing teachers may not be familiar with all of the content and practices outlined in the CCSS. Thus, they may question the need to learn these things in their teacher preparation programs and professional development and may actively resist and reject such instruction. Instructors may need to spend time focusing on the importance of not only a productive disposition toward mathematics,[12] but a recognition of the depth and importance of elementary mathematics, explaining the rationale for its structure and content, and its relationship with the preparation or professional development program.

Responsibility for designing and running elementary teacher preparation programs generally resides in colleges of education, and with faculty members whose

[11] See, e.g., discussion of the use and organization of the blackboard in Lewis, *Lesson Study*, Research for Better Schools, 2002, pp. 97–98.

[12] "Productive disposition" is discussed in the National Research Council report *Adding It Up*, pp. 116–117, 131–133.

primary focus and expertise is not mathematics. These faculty members face increasing pressure to add courses related to English Language Learners, special education, educational policy, assessment, and other contemporary issues, which sometimes leads to the elimination or reduction of mathematics courses for prospective teachers. Faculty may also get push-back from pre-service teachers who do not see the value of the mathematics courses they are required to take. Thus, it is important for those who are concerned with the mathematical preparation of teachers to be in close contact with the faculty who make decisions about the preparation program to educate them about the need for strong mathematical preparation for elementary teachers. Reciprocally, those advocating for the mathematical preparation of teachers need to be well-informed about contemporary issues such as those noted above and thoughtful about how these issues might be addressed in mathematics and mathematics education courses.

Few people trained as mathematicians have thought deeply about how courses for prospective or practicing elementary school teachers might be taught, and there is little support, professional development, or on-the-job training available for them.[13] In some cases, mathematicians do not see the deep study of elementary mathematics content as worthy of college credit. They may try to make the course content "harder" by introducing higher-level mathematics or teach it as a skills course. Or they may ask elementary teachers to take courses such as calculus or other college mathematics courses in lieu of courses on elementary mathematics. In contrast, the content outlined in the previous section shows that there is much to be taught and learned. Colleges and universities need to provide support for those teaching this content to develop their understanding of the manner in which it should be taught.

Practicing teachers may feel overwhelmed by the burdens, mandates, and accountability structures imposed on them by their schools, districts, and states. Teachers in professional development seminars may need some time to communicate with each other about these problems before they turn to more specific thinking about mathematics and its instruction. Professional developers must be sensitive to the pressures that teachers face while also making productive use of valuable time for teachers to think about mathematics and its teaching.[14]

Elementary Mathematics Specialists

Increasingly, school districts have utilized mathematics specialists at the elementary school level.[15] Within their schools, mathematics specialists are regarded as experts. Administrators and other teachers rely upon them for guidance in curriculum selection, instructional decisions, data analysis, teacher mentoring in mathematics, communication with parents, and a host of other matters related to the teaching and learning of mathematics. Depending on location, a specialist

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[13]See discussion of support in Masingila et al., "Who Teaches Mathematics Content Courses for Prospective Elementary Teachers in the United States? Results of a National Survey," *Journal of Mathematics Teacher Education*, 2012.

[14]See, e.g., Schoenfeld, "Working with Schools: The Story of a Mathematics Education Collaboration," *American Mathematical Monthly*, 2009, p. 202.

[15]See Fennell, "We Need Elementary Mathematics Specialists Now, More Than Ever: A Historical Perspective and Call to Action," *National Council of Supervisors of Mathematics Journal*, 2011.

may hold the title elementary mathematics coach, elementary mathematics instructional leader, mathematics support teacher, mathematics resource teacher, mentor teacher, or lead teacher.[16] Specialists serve a variety of functions: mentoring their teacher colleagues, conducting professional development, teaching demonstration lessons, leading co-planning or data teams sessions, observing teachers, or serving as the lead teacher for all of the mathematics classes for a particular group of students.

In several states, specialists and mathematicians collaborate in teaching courses offered for teachers in the specialist's district. Because the specialists remain in their districts, they are able to sustain teachers' learning after the courses. This strategy has been successful in improving student learning.[17]

In 2009, the Association of Mathematics Teacher Educators developed standards for elementary mathematics specialists (EMS), drawing on MET I and other reports.[18] In addition to an understanding of the content in grades K–8, these standards call for EMS to be prepared in the areas of learners and learning (including teachers as adult learners), teaching, and curriculum and assessment. Further, EMS are asked to develop knowledge and skills in the area of leadership as they are often called upon to function in a leadership capacity at the building or district level.

Over a dozen states now offer elementary mathematics specialist certification, and many universities offer graduate degree programs for those wishing to specialize in elementary mathematics education. As with other courses and programs for elementary teachers, mathematicians and mathematics educators have opportunities to work together to develop and teach courses for EMS.

Early Childhood Teachers

Younger children are naturally inquisitive and can be powerful and motivated mathematical learners, who are genuinely interested in exploring mathematical ideas. Currently, there are large disparities in the mathematical abilities of young children. These are linked to socioeconomic status and are larger in the United States than in some other countries. According to the National Research Council report *Mathematics Learning in Early Childhood*, "there is mounting evidence that high-quality preschool can help ameliorate inequities in educational opportunity and begin to address achievement gaps," but "many in the early childhood workforce are not aware of what young children are capable of in mathematics and may not recognize their potential to learn mathematics." Early childhood teachers sometimes hold a variety of beliefs that are not supported by current research.

[16]In general, a math specialist's roles and responsibilities are not analogous to those of a reading specialist.

[17]Examples include the Vermont Mathematics Initiative (a Math Science Partnership), see *Teaching Teachers Mathematics*, Mathematical Sciences Research Institute, 2009, pp. 36–38. A 3-year randomized study in Virginia found that specialists' coaching of teachers had a significant positive effect on student achievement in grades 3–5. The specialists studied completed a mathematics program designed by the Virginia Mathematics and Science Coalition (also a Math Science Partnership) and the findings should not be generalized to specialists with less expertise. See Campbell & Malkus, "The Impact of Elementary Mathematics Coaches on Student Achievement," *Elementary School Journal*, 2011.

[18]*Standards for Elementary Mathematics Specialists: A Reference for Teacher Credentialing and Degree Programs*, Association of Mathematics Teacher Educators, 2009.

These may include "Young children are not ready for mathematics education" or "Computers are inappropriate for the teaching and learning of mathematics."[19]

Mathematics Learning in Early Childhood states:

> Coursework and practicum requirements for early childhood educators should be changed to reflect an increased emphasis on children's mathematics as described in the report. These changes should also be made and enforced by early childhood organizations that oversee credentialing, accreditation, and recognition of teacher professional development programs.

Designers of preparation programs are advised to review their coursework in early childhood mathematics and to prepare teachers in the following areas:

- mathematical concepts and children's mathematical development;
- curricula available for teaching mathematics to young children;
- assessment of young children's mathematical skills and thinking and how to use assessments to inform and improve instructional practice; and
- opportunities to explore and discuss their attitudes and beliefs about mathematics and the effects of those beliefs on their teaching.[20]

Coursework to address these topics satisfactorily will take 6 to 9 semester-hours.

Teachers of Special Populations

The Council for Exceptional Children distinguishes between the roles of teachers "in the core academic subjects" versus other roles that special education teachers play (e.g., co-teaching, helping to design individualized education programs). Similarly, teachers who work with students who are English Language Learners (ELLs) may be teaching mathematics or may be working with students in other capacities (such as developing their language skills and helping them adapt socially). Special education teachers and ELL teachers who have direct responsibility for teaching mathematics (a core academic subject) should have the same level of mathematical knowledge as general education teachers in the subject.

MET II's recommendations for preparation and professional development apply to special education teachers, teachers of ELL students, and any other teacher with direct responsibility for teaching mathematics.

[19]These examples come from Lee & Ginsburg, "Early Childhood Teachers' Misconceptions about Mathematics Education for Young Children in the United States," *Australasian Journal of Early Childhood*, 2009. This article summarizes research in this area and discusses possible sources of such beliefs.

[20]See *Mathematics Learning in Early Childhood*, National Research Council, 2009, pp. 341–343.