CHAPTER 5

Middle Grades Teachers

What mathematics and statistics should future middle grades teachers study to prepare for their careers? What kinds of mathematics coursework and programs will prepare middle grades teachers for teaching mathematics? What professional development experiences will both develop and sustain high quality mathematics teaching in the middle grades? How can mathematicians make valuable contributions to these endeavors? These questions are the topics of this chapter. Coursework in mathematical pedagogy is assumed to be part of a preparation program, but is not discussed in detail.

In this chapter, the term "middle grades teacher" is defined as a teacher who teaches mathematics in grade 6, 7, or 8.[1] The chapter addresses the mathematical knowledge that a middle grades teacher needs to teach, and teach well, the mathematics described for grades 6–8 in the Common Core State Standards for Mathematics (CCSS).

It is important to note that there are distinctions among state requirements for certification to teach mathematics at various grade levels and the requirements found in different teacher preparation programs. Currently 46 states and the District of Columbia provide a license, certificate, or endorsement specific to middle grades. In all but two cases, grade 5 is one of the grades included in the credential.[2]

Many institutions of higher education that prepare teachers do not offer a program specifically and exclusively designed for middle grades teachers of mathematics.[3] Indeed, the majority of middle grades teachers are likely prepared in a program designed as preparation to teach all academic subjects in grades K–8 or in a program to teach mathematics in grades 7–12 or 6–12. Moreover, programs that do offer specific preparation for middle grades often lead to multi-subject certification (such as a certificate to teach mathematics and science), making it challenging for future teachers to take all the mathematics recommended by this report.

Regardless of where middle grades teachers are prepared and how they are certified, it is critical that they have the opportunity to understand the mathematics

Note that the MET II web resources at `www.cbmsweb.org` give URLs for the CCSS, the Progressions for the CCSS, and other relevant information.

[1] As noted in Chapter 3, "Although elementary certification in most states is still a K–6 and, in some states, a K–8 certification, state education departments and accreditation associations are urged to require all grades 5–8 teachers of mathematics to satisfy the 24-hour requirement recommended by this report." Chapters 4 and 5 allow for a period of transition.

[2] See the listing at the Association for Middle Level Education web site.

[3] See, e.g., Tatto & Senk, "The Mathematics Education of Future Primary and Secondary Teachers: Methods and Findings from the Teacher Education and Development Study in Mathematics," *Journal of Mathematics Teacher Education*, 2011, p. 127; *Report of the 2000 National Survey of Mathematics and Science Education*, Horizon Research, p. 16.

in the middle grades from a teacher's perspective. Over time, middle grades mathematics has become more challenging, and the Common Core State Standards outline a significant change in its content, as well as its depth and breadth, for these grades. Many current middle grades teachers, particularly those who teach in grade 6, have elementary certification. Thus, long after it becomes commonplace for future sixth-grade teachers (and many fifth-grade teachers) to earn certification through a middle grades mathematics teacher preparation program, there will be a significant need for content-based professional development opportunities for teachers of mathematics in grades 5 through 8.

Essential Grades 6–8 Ideas for Teachers

This section uses the CCSS as a framework for describing the mathematics that middle grades teachers, both prospective and practicing, should study and know. The CCSS standards for mathematics content are organized into clusters of related standards and the clusters are organized into mathematical domains, which span multiple grade levels (see Appendix B). Brief descriptions of how the mathematics of each domain progresses across grade levels and is connected within or across grades to standards in other domains appear in the Progressions for the CCSS (see the web resources associated with this report).

Because middle grades teachers receive their students from elementary school and prepare them for high school, college courses and professional development opportunities for middle grades teachers should also attend to how the mathematical ideas of the middle grades connect with ideas and topics of elementary school and high school. Thus, courses and professional development will need to devote time to ideas within the K–5 and high school domains (see Chapters 4 and 6).

In this section, essential mathematical ideas are listed for each 6–8 CCSS domain. Teachers need to know these ideas well, but the listings are not intended to be comprehensive. Instructors are encouraged to refer to the CCSS, related progressions, and other references given in the web resources for further details and discussion.

Each list of essential ideas for a domain is followed by a list of related activities that illustrate the ideas and could be used in teacher preparation or professional development.

A given activity may provide opportunities to demonstrate or develop various kinds of expertise described by one or more of the CCSS standards for mathematical practice. These are indicated by the number and heading of the associated standard. For example, "MP 1 Make sense of problems and persevere in solving them" indicates that expertise connected with the first Standard for Mathematical Practice might be used. (The full text for all eight Standards for Mathematical Practice appears as Appendix C of this report.) Note that although a particular activity might provide opportunities to use or increase expertise, instructors should expect to foster engagement in these opportunities. Also, instructors might periodically ask middle grades teachers to review and reflect on the Standards for Mathematical Practice, encouraging teachers to become more familiar with these standards and how they may be achieved in middle grades mathematics.

Ratio and Proportional Relationships (Grades 6–7)[4]

- Reasoning about how quantities vary together in a proportional relationship, using tables, double number lines, and tape diagrams as supports.
- Distinguishing proportional relationships from other relationships, such as additive relationships and inversely proportional relationships.
- Using unit rates to solve problems and to formulate equations for proportional relationships.
- Recognizing that unit rates make connections with prior learning by connecting ratios to fractions.
- Viewing the concept of proportional relationship as an intellectual precursor and key example of a linear relationship.

Illustrative activities:

(1) Examine different ways to solve proportion problems with tables, double number lines, and tape diagrams. Examine common errors students make when solving problems involving ratio and proportion.

MP 2 Reason quantitatively and abstractly.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 5 Use appropriate tools strategically by reasoning with visual models.

(2) Compare and contrast different ways to find values in proportional relationships and in inversely proportional relationships. For example, explain why linear interpolation can be used with proportional relationships but not with inversely proportional relationships.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

The Number System (Grades 6–8)

- Understanding and explaining methods of calculating products and quotients of fraction, by using area models, tape diagrams, and double number lines, and by reading relationships of quantities from equations.
- Using properties of operations (the CCSS term for the field axioms) to explain operations with rational numbers (including negative integers).
- Examining the concepts of greatest common factor and least common multiple.
- Using the standard U.S. division algorithm to explain why decimal expansions of fractions eventually repeat and showing how decimals that eventually repeat can be expressed as fractions.
- Explaining why irrational numbers are needed and how the number system expands from rational to real numbers.

[4]See the Ratio and Proportion Progression for further details, including examples of double number lines and tape diagrams, and discussion of unit rates. In the CCSS, "fractions" refers to non-negative rational numbers in grades 3–5. Note that distinctions made in the CCSS between fractions, ratios, and rates may be unfamiliar to teachers.

Illustrative activities:

(1) Solve fraction division problems using the Group Size Unknown (sharing) perspective and the Number of Groups Unknown (measurement) perspective on division with tape diagrams and double number lines.[5] Use these approaches, as well as the connection between multiplication and division (division can be viewed as multiplication with an unknown factor), to develop rationales for methods of computing quotients of fractions.

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 5 Use appropriate tools strategically.

(2) Explain why rules for adding and subtracting with negative numbers make sense by using properties of operations (e.g., commutativity and associativity of addition and additive inverses) and the connection between addition and subtraction (subtraction can be viewed as finding an unknown addend). Similarly, explain why rules for multiplying and dividing with negative numbers make sense.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 8 Look for and express regularity in repeated reasoning.

(3) Use the standard U.S. division algorithm to explain why the length of the string of repeating digits in the decimal expansion of a fraction is at most 1 less than the denominator. Explain why $0.999\ldots = 1$ in multiple ways.[6]

MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

(4) Prove that there is no rational number whose square is 2.

MP 3 Construct viable arguments and critique the reasoning of others.
MP 7 Look for and make use of structure.

Expressions and Equations (Grades 6–8)

- Viewing numerical and algebraic expressions as "calculation recipes," describing them in words, parsing them into their component parts, and interpreting the components in terms of a context.
- Examining lines of reasoning used to solve equations and systems of equations.
- Viewing proportional relationships and arithmetic sequences as special cases of linear relationships. Reasoning about similar triangles to develop the equation $y = mx + b$ for (non-vertical) lines.

[5]For descriptions of multiplication and division problem types, see the CCSS, p. 89 or the Operations and Algebraic Thinking Progression.

[6]"I don't think it's equal because I think that would be confusing to kids to say that 99 cents can be rounded up to a dollar" and other examples of conceptions that teachers may hold about this equation are given in Yopp et al., "Why It is Important for In-service Elementary Mathematics Teachers to Understand the Equality $.999\ldots = 1$," *Journal of Mathematical Behavior*, 2008. Note that undergraduates may use decimal notation in ways that suggest notions of nonstandard analysis, see Ely, "Nonstandard Student Conceptions About Infinitesimals," *Journal for Research in Mathematics Education*, 2010.

Illustrative activities:

(1) Use tape diagrams as tools in formulating and solving problems and connect the solution strategy to standard algebraic techniques.

MP 1 Make sense of problems and persevere in solving them.
MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.
MP 7 Look for and make use of structure.

(2) Reason about entries of sequences. In particular, determine that and explain why arithmetic sequences are described by formulas of the form $y = mx + b$.

MP 4 Model with mathematics.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

(3) Examine how different types of equations are used for different purposes. (Some equations show the result of a calculation, some equations are to be solved when solving a problem, some equations are used to describe how two quantities vary together, and some equations express identities, such as the distributive property.)

MP 2 Reason abstractly and quantitatively.

Functions (Grade 8)

- Examining and reasoning about functional relationships represented using tables, graphs, equations, and descriptions of functions in words. In particular, examining how the way two quantities change together is reflected in a table, graph, and equation.
- Examining the patterns of change in proportional, linear, inversely proportional, quadratic, and exponential functions, and the types of real-world relationships these functions can model.

Illustrative activities:

(1) Given a graph, tell a story that fits with the graph. Given a story, create a graph that fits with the story.

MP 2 Reason abstractly and quantitatively.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.

(2) Compare and contrast equations, graphs, patterns of change, and types of situations modeled by different relationships. For example, compare and contrast inversely proportional relationships and linear relationships that have graphs with negative slopes. Compare and contrast linear relationships and exponential relationships (including arithmetic sequences and geometric sequences, e.g., contrast repeatedly adding 5 with repeatedly multiplying by 5).

MP 2 Reason abstractly and quantitatively.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

Geometry (Grades 6–8)

- Deriving area formulas such as the formulas for areas of triangles and parallelograms, considering the different height–base cases (including the "very oblique" case where "the height is not directly over the base").
- Explaining why the Pythagorean Theorem is valid in multiple ways. Applying the converse of the Pythagorean Theorem.
- Informally explaining and proving theorems about angles; solving problems about angle relationships.
- Examining dilations, translations, rotations, and reflections, and combinations of these.
- Understanding congruence in terms of translations, rotations, and reflections; and similarity in terms of translations, rotations, reflections, and dilations; solving problems involving congruence and similarity in multiple ways.

Illustrative activities:

(1) Find and explain angle relationships, e.g., the sum of the angles in a 5-pointed star drawn with 10 line segments or the sum of the exterior angles of a shape. Illustrate, informally demonstrate, and prove that the sum of the angles in a triangle is always 180 degrees. Discuss the distinction between an informal demonstration and a proof, as well as ways in which a demonstration can suggest steps in a proof (e.g., tearing off corners and putting them together may suggest the strategy of drawing an auxiliary line).

MP 1 Make sense of problems and persevere in solving them.
MP 3 Construct viable arguments and critique the reasoning of others.
MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

(2) Examine how area and volume change between similar shapes.

MP 5 Use appropriate tools strategically.
MP 7 Look for and make use of structure.
MP 8 Look for and express regularity in repeated reasoning.

Statistics and Probability (Grades 6–8)

- Understanding various ways to summarize, describe, and compare distributions of numerical data in terms of shape, center, and spread.
- Calculating theoretical and experimental probabilities of simple and compound events, and understanding why their values may differ for a given event in a particular experimental situation.
- Developing an understanding of statistical variability and its sources, and the role of randomness in statistical inference.
- Exploring relationships between two variables by studying patterns in bivariate data.

Illustrative activities:

(1) Investigate patterns in repeated random samples or probability experiments to develop a robust understanding of "random."

MP 4 Model with mathematics.
MP 5 Use appropriate tools strategically.

(2) Compare and contrast various measures of center and spread as well as means of calculating them.

MP 4 Model with mathematics.
MP 7 Look for and make use of structure.

(3) Identify sources of variability in data, and draw inferences from analyses of the data.

MP 2 Reason abstractly and quantitatively.
MP 6 Attend to precision.

The Preparation and Professional Development of Middle Grades Teachers

Because the middle grades are "in the middle," it is critical that middle grades teachers be aware of the mathematics that students will study before and after the middle grades. This has significant implications for the preparation of and professional development of middle grades teachers. Middle grades teachers need to be well versed in the mathematics described in Chapter 4, particularly in the domains pertaining to whole numbers and fractions.[7] Moreover, middle grades teachers need to know how the topics they teach are connected to later topics so that they can introduce ideas and representations that will facilitate students' learning of mathematics in high school and beyond. For instance, prospective and practicing middle grades teachers need to be aware of representations, be they drawings, tape diagrams, number lines,[8] or physical models, used in the earlier grades and how those representations may lend themselves to establishing and extending mathematical ideas into the middle grades.[9] For instance, in grades 3–5, area models may be used to represent a product of two fractions, but linear models such as tape diagrams and double number lines are important in the middle grades because they are more readily connected to representations of numbers on the number line and the coordinate plane. Area models may not lead students to successive partitions, which is necessary when thinking about how to partition the interval from 0 to 1 into sixths (partition in half, then partition each half in thirds; or vice versa). On the other hand, area models are used in estimating area under a curve in calculus, and ratio and proportion are intellectual precursors for linear functions.

These examples illustrate the need for middle grades teachers to specialize in mathematics and why their preparation should specifically address mathematics

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[7]It is important for middle grades teachers to have an elementary teacher's perspective on this content because they may need to provide support and instruction for students who have not yet achieved proficiency.

[8]Note that the CCSS use the term "number line diagram" instead of "number line."

[9]Examples of these representations occur in the Progressions for the CCSS.

relevant for teaching grades 5–8. Although this chapter focuses on the mathematics taught in grades 6–8, grade 5 is included here for two reasons. First, teachers receive students from grade 5, thus need to understand the mathematics of grade 5. Second, in most states, middle grades certification includes certification to teach grade 5 and, as recommended in Chapter 3, there should be a transition to the expectation that grade 5 teachers have middle grades certification.

Many, if not most, middle schools offer courses, such as algebra and geometry for high school credit. A middle grades teacher of such courses should have further preparation that goes beyond the recommendations of this chapter, or its professional development equivalent, and be prepared to work closely with high school colleagues in developing appropriate transitions between middle and high school mathematics.

Programs for Prospective Teachers

The mathematics outlined by the Common Core State Standards for grades 6–8 is intellectually challenging and middle grades teachers will require substantial preparation in order to teach it. Initial study of the mathematics for teaching middle grades requires at least 24 semester-hours. At least 15 of these semester-hours should consist of mathematics courses designed specifically for future middle grades teachers that address the essential ideas described in the previous section and in Chapter 4. The remaining 9 semester-hours should include courses that will strengthen prospective teachers' knowledge of mathematics and broaden their understanding of mathematical connections between one grade band and the next, connections between elementary and middle grades as well as between middle grades and high school. This second type of coursework should be carefully selected from mathematics and statistics department offerings. In no case should a course at or below the level of precalculus be considered part of these 24 semester-hours.

Mathematics and statistics courses designed for future middle grades teachers. First and foremost, future teachers need courses that allow them to delve into the mathematics of the middle grades while engaging in mathematical practice as described by the CCSS. The instructors of these courses should model good pedagogy. The courses should be taught with the understanding that the course-takers are future teachers so efforts should be made to connect the mathematics they are learning to mathematics they will teach and challenges they will face when teaching it. These courses should be designed specifically for future middle grades teachers.

Essential ideas that teachers should study in depth and from a teacher's perspective are outlined in the preceding section of this chapter and in Chapter 4. These ideas can be studied in the courses described below. Note that topics which are names of domains in the CCSS (e.g., "ratio and proportional relationships") refer to clusters of ideas described in the corresponding domain and progression for the CCSS.

> *Number and operations* (6 semester-hours). Number and operations in base ten, fractions, addition, subtraction, multiplication, and division with whole numbers, decimals, fractions, and negative numbers. Possible additional topics are irrational numbers or arithmetic in bases other than ten.

Depending on course configuration, some of the topics listed below in the algebra and number theory course might be addressed in a number and operations course.

Geometry and measurement (3 semester-hours). Perimeter, area, surface area, volume, and angle; geometric shapes, transformations, dilations, symmetry, congruence, similarity; and the Pythagorean Theorem and its converse.

Algebra and number theory (3 semester-hours). Expressions and equations, ratio and proportional relationships (and inversely proportional relationships), arithmetic and geometric sequences, functions (linear, quadratic, and exponential), factors and multiples (including greatest common factor and least common multiple), prime numbers and the Fundamental Theorem of Arithmetic, divisibility tests, rational versus irrational numbers. Additional possible topics for teachers who have already studied the above topics in depth and from a teacher's perspective are: polynomial algebra, the division algorithm and the Euclidean algorithm, modular arithmetic.

Statistics and probability (3 semester-hours). Describing and comparing data distributions for both categorical and numerical data, exploring bivariate relationships, exploring elementary probability, and using random sampling as a basis for informal inference.

A necessary prerequisite for the statistics and probability course for middle level teachers is a modern introductory statistics course emphasizing data collection and analysis. This background will allow the course designed for middle grades teachers to emphasize active learning with appropriate hands-on devices and technology while probing deeply into the topics taught in the middle grades, all built around seeing statistics as a four-step investigative process involving question development, data production, data analysis and contextual conclusions.[10]

Other mathematics and statistics courses. This second type of coursework should be carefully selected from mathematics or statistics department offerings that are both useful for and accessible to undergraduates in the institution's middle level education program. It should include:

Introductory statistics. As noted above, this is a recommended prerequisite for the statistics and probability course designed for teachers. The introductory course should have a modern technology-based emphasis with topics that include basic principles of designing a statistical study, data analysis for both categorical and numerical data, and inferential reasoning, much as in the introductory statistics and probability course for high school teachers described in Chapter 6. (In some departments, this might be the same course.)

[10]See the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A PreK–12 Curriculum Framework* of the American Statistical Association.

Other courses might include:

> *Calculus.* Although many institutions require future middle grades mathematics teachers to take the standard first-semester calculus course for engineers, a "concepts of calculus" course might be more useful for those who will be middle level teachers. Such a course could include careful study of the concepts underlying standard topics of calculus (e.g., sequences, series, functions, limits, continuity, differentiation, optimization, curve sketching, anti-differentiation, areas of plane regions, lengths of plane curves, areas of surfaces of revolution, and volumes of solids).
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> *Number theory.* One possibility is a course that focuses on basic number theory results needed to understand the number theoretic RSA cryptography algorithm. As the number theory results are developed, connections to middle level curricula could be emphasized. Proofs should be carefully selected so that they are particularly relevant and accessible to middle level teachers.
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> *Discrete mathematics.* This can offer teachers an opportunity to explore in depth many of the topics they will teach. Possible discrete mathematics topics introduced in this course could include social decision-making, vertex-edge graph theory, counting techniques, matrix models, and the mathematics of iteration. The unifying themes for these topics should be mathematical modeling, the use of technology, algorithmic thinking, recursive thinking, decision-making, and mathematical induction as a way of knowing.
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> *History of mathematics.* A history of mathematics course can provide middle grades teachers with an understanding of the background and historical development of many topics in middle grades (see examples in Chapter 6). Past applications of topics can illustrate their uses in modeling, thus sometimes their historical significance.
>
> *Modeling.* A substantive mathematical modeling course can provide prospective teachers with understanding of the ways in which mathematics and statistics can be applied.

Methods courses. In addition to the mathematics courses described above, prospective middle grades teachers should take two methods courses that address the teaching and learning of mathematics in grades 5–8. At some institutions, it may be possible, and even desirable, to create hybrid courses that integrate the study of mathematics and pedagogy. In these situations, it is still imperative that future teachers complete the equivalent of 24 semester-hours of mathematics.

Professional Development for Practicing Teachers

Teachers of middle grades students must be able to build on their students' earlier mathematics learning and develop a broad set of new understandings and skills to help students meet these more sophisticated mathematical goals. Teaching middle grades mathematics requires preparation *different from* preparation for teaching high school mathematics. Once they begin teaching, middle school teachers need continuing opportunities to deepen and strengthen their mathematical knowledge for teaching, particularly as they engage with students and develop better understanding of their thinking.

Although professional development experiences for middle grades teachers may take a variety of forms, the central focus should be providing opportunities to deepen and strengthen mathematical knowledge in the domains of the CCSS. Many current teachers prepared before the era of the Common Core State Standards will need opportunities to study and learn mathematics and statistics that they have not previously taught. Prior to the CCSS, mathematics in grades 6–8 focused heavily on work with rational numbers (including computational fluency), as well as development of proficiency with geometric measurement (area, surface area, volume) and readiness for algebra (introduction to negative integers, expressions, and equations). In the CCSS, many of these concepts are developed earlier. The shifts in curriculum focus represented by the CCSS (e.g., increased attention to algebra) and the new topics (e.g., transformational approach to congruence), present challenges for many middle grades teachers and underscore the need for professional development.

Professional development for teachers may take many different forms. A group of teachers might work together in a professional learning community, focusing deeply on one topic for a period of time. For example, sixth-grade teachers within a school (or across several schools) might spend a term designing, teaching, and analyzing lessons on expressions and equations using a lesson study format.[11] Or a group of teachers who teach grades 6, 7, and 8 at one school might meet regularly to study how a topic such as proportional reasoning develops across grade levels. A group of teachers might watch demonstration lessons and then meet to discuss the lessons, plan additional lessons, and study the mathematics of the lessons. Math teachers' circles and immersion experiences are additional options.[12] Teachers might also complete mathematics courses specifically designed as part of a graduate program for middle grades mathematics teachers.

Regardless of the format, professional development should engage teachers in mathematics. It should include opportunities for discussing student learning of this mathematics, common student misconceptions, the ways that ideas in the CCSS are related to and build upon one another, and the most useful representations, tools (electronic and otherwise), and strategies for teaching this mathematics to students. However it is organized, as discussed in Chapter 2, the best professional development is ongoing, directly related to the work of teaching mathematics, and focused on mathematical ideas.

Mathematicians and mathematics educators in higher education play an important role in helping to organize, facilitate, or contribute to the professional development of middle school teachers. In doing so, they also have opportunities to think

[11]Lesson study is a process in which teachers jointly plan, observe, analyze, and refine actual classroom lessons.

[12]Math teachers' circles and immersion experiences focus primarily on giving teachers an experience to be learners and doers of mathematics. See Chapter 6 and the web resources for further information and examples.

about undergraduate mathematics teaching and connections between college-level mathematics and K–12 education.

Engaging in mathematical practice. The CCSS standards for mathematical practice describe features of mathematical expertise that learners (including prospective and practicing teachers) develop as they do mathematics. Although these are often discussed separately from mathematical topics, the two should be viewed as inseparable. That is, when doing mathematics, one is engaging in mathematical practice. These features of mathematical practice must be made explicit in preparation and professional development programs. Teachers need to know what they are, and to be able to identify instances in their own work on a particular problem and in children's work, and to be able to think explicitly about when, where, and how these types of expertise would occur in middle grades mathematics.

Using technology and other tools strategically. The tools available for teaching middle grades mathematics include interactive whiteboards and tablets, mathematics-specific technology such as virtual manipulatives, dynamic geometry software, graphing calculators and programs, and an ever-expanding collection of applets, apps, web sites, and multimedia materials. It is essential that prospective and practicing teachers have opportunities to use such tools as they explore mathematical ideas in order to enhance their mathematical thinking, expand the repertoire of technological tools with which they are proficient, and develop an awareness of the limitations of technology. Teachers learn to use technology as a computational tool to perform a calculation or produce a graph or table in order to use the result as input to analyze a mathematical situation. They should also learn to use technology as a problem solving tool, or to conduct an investigation by taking a deliberate mathematical action, observing the consequences, and reflecting on the mathematical implications of the consequences. Teachers must have opportunities to engage in the use of a variety of technological tools, including those designed for mathematics and for teaching mathematics, to explore and deepen their understanding of mathematics, even if these tools are not the same ones they will eventually use with students.

Technology is one of many tools available for learning and teaching mathematics. Others are traditional tools of teaching such as blackboards[13] and geometric models, and newer ones such as patty paper, mirrors, and tangrams.[14] Teachers need to develop the ability to critically evaluate the affordances and limitations of a given tool, both for their own learning and to support the learning of their students. In mathematics courses for teachers, instructors should model successful ways of using tools for teaching, and provide opportunities to discuss mathematical issues that arise in their use.

Challenges in the Education of Middle Grades Teachers

Prospective middle school teachers enter teacher preparation programs with their own views about what it means to know and do mathematics and how it is learned. Because they have chosen to become mathematics teachers, they are likely

[13]See, e.g., discussion of the use and organization of the blackboard in Lewis, *Lesson Study*, Research for Better Schools, 2002, pp. 97–98.

[14]See Kidwell et al., *Tools for Teaching Mathematics in the United States, 1800–2000*, Johns Hopkins University Press, 2008.

to be confident in their abilities to learn mathematics. In this sense, these teacher candidates are more likely to be more similar to pre-service high school teachers than to pre-service elementary teachers. However, their perspective on what it means to know mathematics may be based on their own success in learning facts and procedures rather than on understanding the underlying concepts upon which the procedures are based.

Pre-service middle grades teachers may not be familiar with all of the expectations outlined in the CCSS for middle school students. Thus, they may question the need to learn things in their teacher preparation program that were not part of their own middle grades mathematics.

Although many states offer a distinct certification for middle school teachers (e.g., grades 5–9), other states award middle school endorsements or licenses to teachers together with elementary certification or as part of high school certification (e.g., a teacher is certified to teach grades 7–12). In some states, certified teachers can obtain middle school certification in mathematics by taking and passing an exam such as the Middle School Praxis without taking additional mathematics coursework. Thus, teachers of middle school mathematics are diverse in their mathematical preparation—some have studied the same mathematics as high school teachers; others have completed a few mathematics courses beyond requirements for elementary teachers. Therefore, professional development for middle school mathematics teachers must acknowledge the diversity of background knowledge, both mathematical and pedagogical, that teachers at this level, within a school, district, or state, may have. In any case, the focus of professional development should be on understanding the mathematics outlined in the CCSS for grades 5–8 and instructional strategies to support students in learning it.

Teachers of Special Populations

The Council for Exceptional Children distinguishes between the roles of teachers "in the core academic subjects" versus other roles that special education teachers play (e.g., co-teaching, helping to design individualized education programs). Similarly, teachers who work with students who are English Language Learners (ELLs) may be teaching mathematics or may be working with students in other capacities (such as developing their language skills and helping them adapt socially). Special education teachers and ELL teachers who have direct responsibility for teaching mathematics (a core academic subject) should have the same level of mathematical knowledge as general education teachers in the subject.

MET II's recommendations for preparation and professional development apply to special education teachers, teachers of ELL students, and any other teacher with direct responsibility for teaching mathematics.