Chapter 6 footnotes with hyperlinks

The footnotes from Chapter 6 are listed below and hyperlinked (when possible) to the references cited.

Many of the documents cited are freely available. National Research Council reports such as *Adding It Up* can be read on-line. They can be downloaded without charge as can documents from the Conference Board of the Mathematical Sciences and the Council of Chief State School Officers. In some cases, cited portions of documents can be seen via Google Books.

Mathematics education research journal articles are likely to require a subscription. At many academic institutions, these journals will be accessible via institutional subscription. Attempts to access a JSTOR link without such a subscription will get the response "Cannot download the information you requested."

> Note that the MET II web resources at www.cbmsweb.org give URLs for the CCSS, the Progressions for the CCSS, and other relevant information.

1. Translation of the third edition, Macmillan, 1932, p. 1.

2. This line of research and its limitations are discussed in more detail in Chapter 2.

3. "On the Education of Mathematics Majors" in *Contemporary Issues in Mathematics Education*, Mathematical Sciences Research Institute, 1999, p. 13.

4. See Recommendation 13 of *National Task Force on Teacher Education in Physics: Report Synopsis*, American Association of Physics Teachers, the American Physical Society, & the American Institute of Physics, 2010.

5. For examples, see Wu, "Phoenix Rising," *American Educator*, 2011.

6. The CCSS standards for high school include standards marked with a +, indicating standards that are beyond the college- and career-ready threshold.

7. From a modern viewpoint, this is an application, but the notion of group arose in this context. See Grattan-Guinness's discussion of "irresolving the quintic" in *The Rainbow of Mathematics: A History of the Mathematical Sciences*, Norton, 1997.

8. See, e.g., Howe, "The Secret Life of the $ax + b$ Group" in the web resources.

9. These and other ideas are listed in Kleiner's "The Teaching of Abstract Algebra: An Historical Perspective" in *Learn From the Masters!*, MAA, 1995.

10. See Smith & Karpinski, *Hindu-Arabic Numerals*, Ginn and Company, 1911, pp. 136–137.

11. This distinction is illustrated by $x2$ and x vs. sq and rt (or square and root).

12. For details of previous and subsequent notations, see Cajori, *A History of Mathematical Notations*, Dover, 1993. A similarity between base-ten notation and symbolic algebra is that they are "action notations" in which computations can occur, rather than "display notations" that only record results. See Kaput, "Democratizing Access to Calculus," *Mathematical Thinking and Problem Solving*, Erlbaum, 1994, p. 101.

13. This description is based on the University of California at Berkeley courses 151, 152, 153.

14. This description is based on the University of California, Santa Barbara courses 101A-B, 102A-B, 103.

15. Rotman's *Journey Through Mathematics* has been used for such a course.

16. Part of an eighth grade standard is: "Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane."

17. For example, see the reports of Focus on Mathematics (a Math Science Partnership). Comments from teachers include: "Study groups have made 'asking the next question' a much more intriguing mathematical exploration than I previously had imagined or realized I could access," *Focus on Mathematics Summative Evaluation Report 2009*, p. 29.