

The Mathematical Education of High School Teachers, Digital Resource 5.1
Working with Relations¹

Task

A fish is swimming from home toward food, and wants to avoid disturbing a particularly ferocious predator. The following figure represents a simplification of the fish's path to food.



1. Create a graph that relates the fish's distance from home to its distance from food.
2. Create a graph that relates the fish's distance from home to its distance to the predator.
3. What would a more realistic path look like? How would the resulting graph compare to the one you created in #2?

¹ A variation of a task from Paoletti et al. (2016).

Commentary

This example shows a context in which teachers are asked to consider the path of a fish headed to its feeding ground, where a predator stands in the way. One simple solution is for the fish to just maintain a minimum safe distance from the predator, represented by a semi-circle, as shown in the figure in the task in the previous section. Possible solutions for this approach are given in the following section.

Teachers might consider what happens as the safe distance increases or decreases, perhaps adding a parameter as shown in this [Geogebra activity](#).

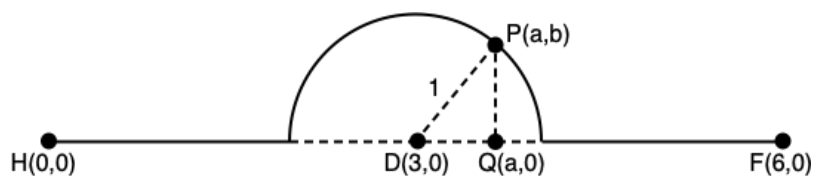
Teachers might further consider other paths the fish may take, given that this is a very straightforward model, and consider how the graphs would change.

The follow-up discussion might focus on how more realistic paths for the fish might be modeled and how they would impact the graphs. For example, they might use [this Geogebra activity](#) to adjust the path to explore other scenarios. They could further discuss what would happen if the fish backtracks or circles around the food, rather than proceeding in a strictly left-to-right manner, as in this [Geogebra activity](#), which provides the ability to draw an arbitrary path.

Teachers should also discuss how the task might be adapted for use in a high school mathematics classroom.

Solutions

1. Let h be the distance from home, $H(0,0)$, and f be the distance from the food, $F(6,0)$. Also, assume the curvilinear portion of the path is a semi-circle with center $D(3,0)$.

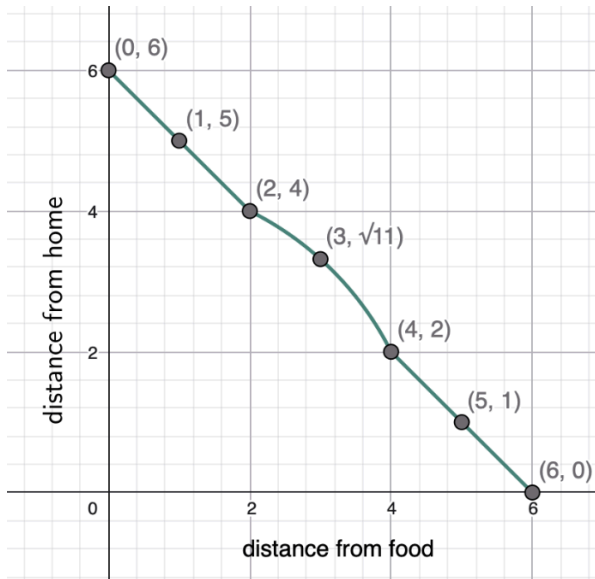


The relationship for the non-curvilinear portion of the path is simply $h = 6 - f$

Choose $P(a,b)$ on the semi-circle. Then $b = \sqrt{1 - (3 - a)^2}$

We can then compute $h = \sqrt{a^2 + b^2} = \sqrt{6a - 8}$ and $f = \sqrt{(6 - a)^2 + b^2} = \sqrt{28 - 6a}$, using the Pythagorean theorem.

Solving for a in terms of h , $a = \frac{h^2 + 8}{6}$ and so $f = \sqrt{20 - h^2}$, which is the relationship between f and h on the semi-circle. Thus, the graph looks as follows:



2. Continuing on with the definitions from above, also let $d = HD$.
 For the non-curvilinear portion of the path, $d = |h - 3|$.
 For the curvilinear portion, $d = 1$, the radius of the circle.
 Thus, the graph will look as shown below.

