*The Mathematical Education of High School Teachers, Digital Resource 5.2*

# Reasoning from Covariation and Correspondence[1]

## Task

A teacher asked students to write the equation of a linear function that could be represented by the values shown in the following table:

| $x$ | $y$ |
|---|---|
| 1 | 6 |
| 2 | 10 |
| 3 | 14 |
| 4 | 18 |

Students gave different explanations for how they found the same equation, as follow:

> **Student A:** Each time the value of x goes up by 1, the value of y goes up by 4, so the slope is 4. And if x goes down by 1, then y will have to go down by 4. So the y-intercept is 2. That means the equation is y = 4x + 2.
>
> **Student B:** For this function, I saw that you can always multiply the value of x by 4 and then add 2 to get the value of y, so the equation is y = 4x + 2.

1. Compare the reasoning used by the two students. How are they the same or different?

2. How might you make connections between the reasoning used by the two students?

3. As a teacher, how might you use their solutions to guide the class' understanding of linear functions?

[1] Task from Howell et al. (2021).

## Commentary

This task is designed to highlight two views of functions and relations: the *covariation* view used by Student A and the *correspondence* view used by Student B. As stated in the *METIII* document (Conference Board of Mathematical Sciences [CBMS], forthcoming):

> By a covariation view, we mean understanding how changing the value of one quantity impacts the value of the other quantity. A covariation view supports investigating relationships among variables in mathematical modeling, reasoning computationally, and reasoning statistically.

In contrast,

> A correspondence view treats functions and relations in terms of individual inputs and the images to which they are associated (in other words, to which they correspond). This perspective connects secondary mathematics to areas of higher-level mathematics such as algebra, discrete mathematics, real analysis, and topology.

In the context of this problem, Student A recognizes the constant rate of change over equal intervals, a covariation perspective, while Student B looks for the pattern in what values are assigned to different inputs, recognizing that the same value always works. Both of the views are important and should be attended to.

This task might be productively used to introduce teachers to the covariation and correspondence views of functions and relations. Rather than beginning by the lessons defining the terms, teachers should have an opportunity to analyze the student reasoning inherent in each view. Furthermore, they should also have an opportunity to see how these perspectives connect. At the end of the activity, the formal terms could be introduced, highlighting how the terms match the kind of thinking: whether it is examining how the variables vary in relation to each other (e.g., how they covary) or looking at what outputs are assigned to a given input (e.g., corresponding values.)

## Solution

1. Student A is focusing on how the values grow, while Student B is focusing on the pattern in the outputs. While they end up at the same result, their paths in getting to that result are quite different.

2. Student A notices the constant rate of change and uses that to work back to the value of the function at 0 by subtracting 4. Note that repeatedly For a given whole number x, you will need to repeatedly add the same value, 4 in this case, x times, which is really the same as multiplying 4 by $x$. Thus, the formula should be 2 + 4x. (We could do additional work to show this works for non-whole numbers.)

3. The teacher might have each student present their work to the class, then ask the other members of the class to discuss each approach and compare them in order to see both views.. The usefulness and validity of each approach should be emphasized.